# Mixtures of maximally entangled pure states

M.M. Flores*, E.A. Galapon**

*Theoretical Physics Group, National Institute of Physics, University of the Philippines, Diliman, Quezon City*

**Abstract**

We study the conditions when mixtures of maximally entangled pure states remain entangled. We found that the resulting mixed state remains entangled when the number of entangled pure states to be mixed is less than or equal to the dimension of the pure states. For the latter case of mixing a number of pure states equal to their dimension, we found that the mixed state is entangled provided that the entangled pure states to be mixed are not equally weighted. We also found that one can restrict the set of pure states that one can mix from in order to ensure that the resulting mixed state is genuinely entangled. Also, we demonstrate how these results could be applied as a way to detect entanglement in mixtures of the entangled pure states with noise.

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## 1. Introduction

One of the outstanding problems in the theory of quantum entanglement, which is of paramount importance when using entanglement as a resource for quantum information processing is its detection [1]. In other words, given an arbitrary quantum state, the problem of detection asks whether the state is entangled or not. The difficulty of the problem of entanglement detection is reflected in the abundance of separability criteria in literature [1, 2], some involving extremizations and working only for some special cases. The problem is even more pronounced in the case of mixed states, where the typical recourse would be to construct convex roof extensions of existing measures for pure states, something which is hard to compute in general [5–7]. Hence, it would be very helpful if we can come up with classes of mixed states which are entangled, thus saving us from having to apply an entanglement detection scheme should a given state belong to these classes. One of the ways to do this is by considering mixtures of maximally entangled pure states and studying the conditions under which the resulting mixed state is guaranteed to be entangled.

In this paper, we introduce a class of entangled mixed states by mixing distinct $N$-partite entangled pure states having equal dimensions $d$. We are motivated by the fact that although a convex combination of separable states is again separable, the mixture of entangled pure sates is not necessarily entangled as can be easily seen by mixing two distinct maximally entangled Bell states of equal weights [8]. This fact prompted us to particularly focus on the mixtures of *maximally* entangled pure states. Here, we define maximally entangled pure states to be those states where all the one-qudit reduced density matrices are maximally mixed. That is, a pure state $|\phi\rangle \in \mathcal{H}$ is maximally entangled provided that $\mathrm{tr}_{\mathcal{H}/\mathcal{H}_k}|\Phi\rangle\langle\Phi| = \frac{\mathbb{I}_k}{d}$ where $\mathbb{I}_k$ is the identity matrix in $\mathcal{H}_k$ and $d$ is the dimension of $\mathcal{H}_k$. Note that some authors require additional criteria for maximal entanglement depending on the measure of entanglement being utilized [25, 26]. One such criterion is that the state should be maximally (bipartite) entangled for all possible bipartitions since the measure of entanglement being used is based on the average bipartite entanglement [11, 12, 29]. Our choice of definition

*Principal corresponding author
**Corresponding author
*Email addresses:* mflores@nip.up.edu.ph (M.M. Flores), eric.galapon@gmail.com (E.A. Galapon)