Annals of Physics 372 (2016) 309–319

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Real-time evolution of strongly coupled fermions driven by dissipation

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A R T I C L E I N F O

Article history:
Received 2 February 2016
Accepted 31 May 2016
Available online 6 June 2016

Keywords:
Open systems
Fermion systems
Dissipative systems
Real-time dynamics
Lindblad operators
Non-equilibrium

A B S T R A C T

We consider the real-time evolution of a strongly coupled system of lattice fermions whose dynamics is driven entirely by dissipative Lindblad processes, with linear or quadratic quantum jump operators. The fermion 2-point functions obey a closed set of differential equations, which can be solved with linear algebra methods. The staggered occupation order parameter of the t-V model decreases exponentially during the dissipative time evolution. The structure factor associated with the various Fourier modes shows the slowing down of low-momentum modes, which is due to particle number conservation. The processes with nearest-neighbor-dependent Lindblad operators have a decay rate that is proportional to the coordination number of the spatial lattice.

1. Introduction

Understanding the real-time dynamics of large strongly coupled quantum systems is a challenge that affects many areas of physics. In particular, simulations of the real-time path integral on classical computers suffer from severe complex action problems, which prevent the application of importance sampling underlying quantum Monte Carlo [1]. Since the unitary time evolution of large isolated quantum systems may lead into paradoxical Schrödinger-cat-like states, it seems unlikely that their

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http://dx.doi.org/10.1016/j.aop.2016.05.019

behavior can be captured by classical computation. Quantum simulators, i.e. special purpose digital [2] or analog [3] quantum computers, are emerging as promising new tools that, thanks to their quantum hardware, do not suffer from sign or complex action problems. They can thus be used to address systems for which no "classical" solution of the sign problem has been found. Quantum simulators have been constructed in atomic and condensed matter physics [4–9], and more recently also in a particle physics context [10–18]. Since quantum simulators are not yet readily available for most systems, it is interesting to ask what aspects of real-time quantum dynamics can be captured by simulations using classical computers. First of all, a lot of progress has been made for gapped systems with moderate entanglement in one spatial dimension. In this case, the matrix product states underlying the density matrix renormalization group (DMRG) [19,20] provide a basis for simulating the real-time evolution over moderate time intervals [21–27]. For large higher-dimensional quantum systems, on the other hand, no unbiased computational method exists for addressing real-time dynamics.

Purely dissipative dynamics driven by a Lindblad process, in which the Hamiltonian is discarded, plays an important role for state preparation of ultracold atom systems [28–38]. Dissipation can also serve as a resource for quantum computation [39–43] and for entanglement generation [44]. Recently, some severe sign problems have been solved for strongly interacting 2-dimensional quantum spin systems that are entirely driven by their coupling to a dissipative environment [45]. The Lindblad processes simulated so far can also be interpreted as sporadic randomized measurement processes, for example, of the total spin of a nearest-neighbor spin pair. A Lindblad dynamics with Hermitean quantum jump operators, which is equivalent to a measurement process, ultimately leads to a featureless infinite temperature density matrix. In spite of this, the corresponding heating process leads far out of thermal equilibrium and is interesting to investigate. In particular, depending on the dissipative process, some physical quantities like the total magnetization or staggered magnetization may be conserved. In that case, the thermalization of certain magnetization Fourier modes is slowed down [46], and the transport of the conserved quantity is governed by a diffusion process. The flow of magnetization or staggered magnetization between a ferro- and an antiferromagnetic reservoir, occupying two initially separated parts of the volume, has been studied in [47].

In this paper, we investigate Lindblad processes for fermions, with linear or quadratic quantum jump operators. Again, we study purely dissipative dynamics driven by the interaction with an environment, not by a Hamiltonian. Even in Euclidean time, fermion simulations are often challenging. Due to Fermi statistics, negative signs enter the fermion path integral and may prevent the application of importance sampling. In such cases, a severe fermion sign problem arises. In specific cases, in particular at half-filling, integrating out the fermions may lead to a non-negative fermion determinant, such that Monte Carlo is applicable. In other cases, for example, in the repulsive Hubbard model away from half-filling or in lattice QCD at non-zero baryon density, a severe sign problem prevents numerical investigations from first principles. Still, in some interesting cases, severe fermion sign problems have been solved, for example, using the meron-cluster algorithm [48] or the fermion bag approach [49–52]. Fermion simulations in real time are particularly challenging, because one encounters both the fermion sign problem and the complex weight problem characteristic for real-time processes. It is known that for some Hermitean jump operators, analytic expressions for some correlations can be found in bosonic and fermionic systems and criteria for when this is possible have been developed [53]. Steady states for some of the correlations, specifically for dissipative one-dimensional noninteracting spin systems, have been found [54–57]. Here we identify specific fermionic Lindblad processes, with linear or quadratic quantum jump operators, for which analytic expressions for the real-time evolution of certain observables may be obtained, and we analyze their behavior. As an example, we use the Hamiltonian of the t-V model to generate a correlated initial fermion density matrix, but the results are more general and would be applicable to any pure state or a more general density matrix.

At low temperature, the t-V Hamiltonian leads to a staggered occupation of fermions on a bipartite lattice. The real-time process, which is driven entirely by the Lindblad dynamics (and not by the Hamiltonian), leads to an exponentially fast destruction of the staggered occupation order. In the case of quadratic quantum jump operators, we investigate the dependence of the exponential decay rate on the lattice geometry. As with the quantum spin systems studied in [45–47], the thermalization of some Fourier modes of the fermion density is slowed down due to fermion number conservation.