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Quantum dynamics of relativistic bosons through nonminimal vector square potentials

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HIGHLIGHTS

- DKP bosons in a nonminimal vector square potential are studied.
- Spin zero and spin one bosons have the same results.
- The Schiff–Snyder–Weinberg effect is observed.

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ABSTRACT

The dynamics of relativistic bosons (scalar and vectorial) through nonminimal vector square (well and barrier) potentials is studied in the Duffin–Kemmer–Petiau (DKP) formalism. We show that the problem can be mapped in effective Schrödinger equations for a component of the DKP spinor. An oscillatory transmission coefficient is found and there is total reflection. Additionally, the energy spectrum of bound states is obtained and reveals the Schiff–Snyder–Weinberg effect, for specific conditions the potential lodges bound states of particles and antiparticles.

1. Introduction

The pioneering works of Duffin [1], Kemmer [2,3] and Petiau [4] (DKP) gave rise to a rich formalism, similar to Dirac theory, able to describe interactions of spin 0 and spin 1 bosons. Various additional couplings, impossible to be explored in conventional Klein–Gordon and Proca equations, gave rise to a large area of physical applications such as describing the scattering of mesons by nuclei [5–7], the dynamics of bosons in curved space–time [8] and noninertial effect of rotating frames [9],

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thermodynamic properties of bosons in noncommutative plane [10], all works involving spin 0 systems. Vector bosons in the expanding universe [11] and in an Aharonov–Bohm potential [12] are examples of applications to spin 1 systems. The Bose–Einstein condensate [13,14], very special relativity [15], among others works, are applications for both spin systems.

The interest of one-dimensional potentials in DKP formalism has increased significantly in recent decades, because the simplicity of equations obtained provides great support for studying physical systems in higher dimensions. Among the potentials used, we can highlight the double-step potential [16,17], the DKP oscillator [18,19], the inversely linear background [20], the mixed minimal–nonminimal vector cusp potential [21].

In this spirit, the purpose of this article is to address the problem of scalar and vector bosons subjected to a nonminimal vector square (well and barrier) potential in the DKP formalism. We obtain a transmission coefficient that shows oscillatory behavior, where we can observe the resonance tunneling. Additionally, we obtain the energy spectrum of bound states by a simple and transparent way. We show that the eigenenergies obtained have great similarity to the problem of fermions in the same potential, already explored in the literature [22].

2. A review of DKP equation

The DKP equation for a free boson is given by [2] (in natural units, $\hbar = c = 1$)

$$\left(i\beta^{\mu}\partial_{\mu} - m\right)\psi = 0 \tag{1}$$

where the matrices β^{μ} satisfy the algebra $\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\lambda\nu}\beta^{\mu}$ and the metric tensor is $g^{\mu\nu} = \text{diag}\,(1, -1, -1, -1)$. The conserved four-current is given by $J^{\mu} = \bar{\psi}\beta^{\mu}\psi/2$ where the adjoint spinor $\bar{\psi}$ is given by $\bar{\psi} = \psi^{\dagger}\eta^{0}$ with $\eta^{0} = 2\beta^{0}\beta^{0} - 1$. The correct use of nonminimal interactions in the DKP equation can be found in [23], where the continuity equation implies in conserved physical quantities.

With nonminimal vector interactions, the DKP equation can be written as [24],

$$\left(i\beta^{\mu}\partial_{\mu} - m - i[P, \beta^{\mu}]A_{\mu}\right)\psi = 0 \tag{2}$$

where P is a projection operator ($P^2 = P$ and $P^{\dagger} = P$) in such a way that $\bar{\psi}[P, \beta^{\mu}]\psi$ behaves like a vector under a Lorentz transformation as $\bar{\psi}\beta^{\mu}\psi$ does. If the potential is time-independent one can write $\psi(\vec{r}, t) = \phi(\vec{r})\exp(-iEt)$, where E is the energy of the boson, the DKP equation becomes

$$\left[\beta^{0}E + i\beta^{i}\partial_{i} - \left(m + i[P, \beta^{\mu}]A_{\mu}\right)\right]\phi = 0. \tag{3}$$

2.1. Scalar sector

For the scalar bosons, we use the representation for the β^{μ} matrices given by [25]

$$\beta^{0} = \begin{pmatrix} \theta & \overline{0} \\ \overline{0}^{T} & \mathbf{0} \end{pmatrix}, \qquad \beta^{i} = \begin{pmatrix} \widetilde{0} & \rho_i \\ -\rho_i^{T} & \mathbf{0} \end{pmatrix}, \quad i = 1, 2, 3 \tag{4}$$

where

$$\begin{aligned} \theta &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \rho_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \rho_2 &= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \rho_3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{5}$$

$\overline{0}$, $\widetilde{0}$ and $\mathbf{0}$ are 2×3, 2×2 and 3×3 zero matrices, respectively, while the superscript T designates matrix transposition. Here the projection operator can be written as [26] $P = \left(\beta^{\mu}\beta_{\mu} - 1\right)/3 = \text{diag}\,(1, 0, 0, 0, 0)$. In this case P picks out the first component of the DKP spinor. The five-component spinor can be written as $\psi^{T} = (\phi_1, \ldots, \phi_5)$ in such a way that the time-independent DKP equation