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Hidden global conformal symmetry without Virasoro extension in theory of elasticity

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ABSTRACT

The theory of elasticity (a.k.a. Riva–Cardy model) has been regarded as an example of scale invariant but not conformal field theories. We argue that in $d = 2$ dimensions, the theory has hidden global conformal symmetry of $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ without its Virasoro extension. More precisely, we can embed all the correlation functions of the displacement vector into a global conformal field theory with four-derivative action in terms of two scalar potential variables, which necessarily violates the reflection positivity. The energy–momentum tensor for the potential variables cannot be improved to become traceless so that it does not show the Virasoro symmetry even with the existence of global special conformal current.

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In recent years, the question under which condition scale invariance implies conformal invariance has gained some renewed interest (e.g. [1–6]).[1] Due to the revival of conformal bootstrap in $d > 2$ dimensions (see e.g. [8–10] and references therein), the question has become not only academic but also practically important to reveal the nature of the renormalization group fixed point.

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1 For a review and earlier reference, see [7].

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One interesting example of scale invariant but not conformal invariant field theories discussed in the literature is the theory of elasticity [11]. For a displacement vector v_μ, we have the (Euclidean) action

$$S = \int d^d x \left(-\frac{1}{4}(\partial_\mu v_\nu - \partial_\nu v_\mu)^2 - \frac{\kappa}{2}(\partial_\mu v^\mu)^2 \right). \tag{1}$$

Originally, Riva and Cardy [12] examined the model in $d = 2$ dimensions. In general d-dimensions with generic κ, the theory has been regarded as an example of scale invariant but not conformal invariant field theories [13].[2] Up to improvement terms, the symmetric energy–momentum tensor is given by

$$\begin{aligned} T_{\mu\nu} = {} & \frac{\delta_{\mu\nu}}{4}(\partial_\rho v_\sigma - \partial_\rho v_\sigma)^2 - (\partial_\mu v^\rho - \partial^\rho v_\mu)(\partial_\rho v_\nu - \partial_\nu v_\rho) \\ & + \kappa(v_\mu \partial_\nu(\partial^\rho v_\rho) + v_\nu \partial_\mu(\partial^\rho v_\rho) - \delta_{\mu\nu}(v_\rho \partial^\rho(\partial^\sigma v_\sigma) + (\partial^\rho v_\rho)^2/2)). \end{aligned} \tag{2}$$

Let us recall that the condition of the scale invariance is that the trace of the symmetric energy–momentum tensor is given by divergence of the Virial current J_μ [14,15].

$$T^\mu_\mu = \partial^\mu J_\mu. \tag{3}$$

The condition for the global conformal invariance is that the Virial current is further rewritten as

$$J_\mu = \partial^\nu L_{\mu\nu}, \tag{4}$$

with another symmetric tensor $L_{\mu\nu}$. Indeed, if this is the case, we can construct a special conformal current

$$J^{(k)}_\mu = T_{\mu\nu}(2k_\rho x^\rho x^\nu - x^2 k^\nu) - 2k_\rho x^\rho \partial^\nu L_{\mu\nu} + 2k^\nu L_{\mu\nu} \tag{5}$$

that is conserved for a constant vector k_μ in any space-time dimensions (including $d = 2$).

In $d = 2$ dimensions, we typically observe that the global conformal symmetry of $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ enjoys the Virasoro extension. The precise condition for the Virasoro extension is that the energy–momentum tensor is improved to become traceless. For this, we further need to require [16, 17]

$$L_{\mu\nu} = \delta_{\mu\nu} L \tag{6}$$

for a scalar operator L so that we can construct the improved energy–momentum tensor that is traceless:

$$\begin{aligned} \tilde{T}_{\mu\nu} &= T_{\mu\nu} + (\partial_\mu \partial_\nu - \delta_{\mu\nu}\Box)L \\ \tilde{T}^\mu_\mu &= 0. \end{aligned} \tag{7}$$

In $d = 2$ dimensions, it then means that we can construct the holomorphic energy–momentum tensor $T_{zz}(z)$ in the complex coordinate $z = x_1 + ix_2$, leading to the infinite number of conserved current $j^{(\epsilon)}(z) = \epsilon(z)T_{zz}(z)$ for any holomorphic function $\epsilon(z)$. Note that this condition is specific to the $d = 2$ dimensions, and there is a niche possibility of having global conformal invariance without the Virasoro extension. As we will discuss at the end of this paper, the situation is rare, and it is only possible at the sacrifice of reflection positivity.

From the viewpoint of the Weyl invariance in the curved background, the special feature in $d = 2$ dimension comes from the fact that the Ricci tensor and Ricci scalar are related as $R_{\mu\nu} = Rg_{\mu\nu}/2$, so the curvature interaction $\int d^2x\sqrt{g}R_{\mu\nu}L^{\mu\nu}$ does not contain the symmetric traceless part. Therefore, in $d = 2$ dimensions, the improvement term is able to remove the trace part of the symmetric tensor $L_{\mu\nu}$ but not the traceless part.

[2] In any dimensions, at $\kappa = 1$, we have topologically twisted conformal symmetry. In $d > 2$ dimensions, we have untwisted (i.e. physical) conformal symmetry at $\kappa = (d-4)/d$.

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