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# Entropy bound of horizons for accelerating, rotating and charged Plebanski–Demianski black hole

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ABSTRACT

We first review the accelerating, rotating and charged Plebanski–Demianski (PD) black hole, which includes the Kerr–Newman rotating black hole and the Taub-NUT spacetime. The main feature of this black hole is that it has 4 horizons like event horizon, Cauchy horizon and two accelerating horizons. In the non-extremal case, the surface area, entropy, surface gravity, temperature, angular velocity, Komar energy and irreducible mass on the event horizon and Cauchy horizon are presented for PD black hole. The entropy product, temperature product, Komar energy product and irreducible mass product have been found for event horizon and Cauchy horizon. Also their sums are found for both horizons. All these relations are dependent on the mass of the PD black hole and other parameters. So all the products are not universal for PD black hole. The entropy and area bounds for two horizons have been investigated. Also we found the Christodoulou–Ruffini mass for extremal PD black hole. Finally, using first law of thermodynamics, we also found the Smarr relation for PD black hole.



## 1. Introduction

In 1981, Bekenstein [1] proposed the universal bound on the entropy of a macroscopic object of maximal radius $R$ bearing energy $E$ in the form $S \leq \frac{2\pi ER}{\hbar}$. But the derivation of the entropy bound was criticized by Unruh, Wald and Pelath [2–4]. Bekenstein refuted their criticism and showed that

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buoyancy is so negligible such that it does not spoil the entropy bound derivation [5,6]. In various occasions, this type of bound has been found in some literatures by the same author [7–9]. In 1992, Zaslavskii [10] modified the entropy bound by incorporating the charge of a black hole. After that Bekenstein and Mayo [11], Hod [12,13] and Linet [14] obtained an upper entropy bound for a charged Reissner–Nordstrom black hole, in the form $S \le \frac{2\pi}{\hbar}\left(E^2R - \frac{e^2}{2}\right)$, where $e$ is the electric charge of the black hole. This result agrees to an earlier finding by Zaslavskii [10] in another context. However, the fact that this entropy bound for a charged system is necessary to uphold the GSL has been challenged as well [15]. A tighter bound on entropy for objects with angular momentum has also been derived by Hod [12]. Referring to Hojman and Hojman's [16] integrals of motion for a neutral object with spin $s$ moving on a Kerr black hole background, Hod [12] obtained the entropy bound $S \le \frac{2\pi}{\hbar}\left(E^2R^2 - s^2\right)^{\frac{1}{2}}$. Wang and Abdalla [17] studied the entropy bound for a spinning object falling into anti de Sitter (AdS) black holes including $(3+1)$-dimensional Kerr-AdS black holes and $(2+1)$-dimensional Banados–Teitelboim–Zanelli (BTZ) black holes. The entropy bounds for these black holes are identical with the Kerr black hole. Linet [18] and Qiu et al. [19] obtained the upper bound of the entropy on the more general Kerr–Newman black hole as in the form $S \le 2\pi\left(\mu R - \frac{e^2}{2}\right)$, where $\mu$ is related to total energy, angular momentum, charge, radius and mass of the black hole. Jing [20] obtained the Cardy–Verlinde formula and the entropy bounds in Kerr–Newman-$AdS_4$/$dS_4$ black hole. The Bekenstein–Verlinde-like entropy bound in Kerr–Newman-$AdS_4$ black hole [20] is $S \le \frac{2\pi}{n}\left(ER - \frac{e^2}{2H}\right)$ and the entropy bound in Kerr–Newman-$dS_4$ black hole [20] is $S \le \frac{2\pi}{H}\left(MR - \frac{e^2}{2}\right)$, where $H = 1 - \frac{a^2}{l^2}$, $a$ is the angular momentum and $l$ is related to the cosmological constant $\Lambda$.

The product of horizon areas or the entropy product of horizons of black hole is very important tool in the study of black hole physics. The two horizons of the black hole are namely inner/Cauchy horizon $(\mathcal{H}^-)$ and outer/event horizon $(\mathcal{H}^+)$. Now it is known that the Cauchy horizon $(\mathcal{H}^-)$ is an infinite blue-shift surface, but the event horizon $(\mathcal{H}^+)$ is an infinite red-shift surface [21]. For stationary axially symmetric black holes, the entropy product of horizons is often independent of the mass of the black hole [22–26]. Such products depend on the charge and angular momentum of the black hole. In some cases, this relation may be depend on the mass of the black hole [27–29]. So the entropy sum and other thermodynamic relations have been studied by some authors [30–34]. In some cases, these relations may be independent of black hole mass and some cases, these depend on black hole mass. The regular axisymmetric and stationary spacetime of an Einstein–Maxwell system with surrounding matter have a regular Cauchy horizon $(\mathcal{H}^-)$, which always occurs inside the event horizon $(\mathcal{H}^+)$ if and only if the angular momentum $J$ and charge $Q$ of the black hole do not vanish simultaneously. In this case, the Cauchy horizon $(\mathcal{H}^-)$ becomes singular and tends to a curvature singularity when $J$ and $Q$ tend to zero [35–37]. In Boyer–Lindquist coordinates, the existence of Cauchy horizon describes that the stationary and axisymmetric Einstein–Maxwell electro-vacuum equations are hyperbolic in the interior vicinity of the event horizon $(\mathcal{H}^+)$. The two horizons $\mathcal{H}^+$ and $\mathcal{H}^-$ describe the future and past boundary of this hyperbolic region. If the Cauchy horizon exists i.e., if $J$ and $Q$ do not vanish simultaneously, then the product of the entropy (area) of the horizons $\mathcal{H}^\pm$ for the Kerr–Newman black hole is independent of the mass of the black hole, but depends on the angular momentum $J$ and the charge $Q$ explicitly [30].

Based on the entropy product and entropy sum, very recently, Xu et al. [38] obtained entropy (area) bounds of event horizon $(\mathcal{H}^+)$ and Cauchy horizon $(\mathcal{H}^-)$ for Kerr black hole, Kerr–Newman black hole in Gauss Bonnet gravity and Kerr–Taub-NUT black hole. They have actually taken the Penrose-like inequality for the upper area bound of the event horizon $(\mathcal{H}^+)$. They have also found that (i) the electric charge $Q$ diminishes the physical bound of entropy (area) for event horizon $(\mathcal{H}^+)$, while it enlarges that for Cauchy horizon $(\mathcal{H}^-)$; (ii) the angular momentum $J$ enlarges them for Cauchy horizon $(\mathcal{H}^-)$, while it does nothing with that for event horizon $(\mathcal{H}^+)$ and (iii) the NUT charge $n$ always enlarges them for both event horizon $(\mathcal{H}^+)$ and Cauchy horizon $(\mathcal{H}^-)$. With the ideas of their work, we now formulate the entropy bounds on event horizon $(\mathcal{H}^+)$ and Cauchy horizon $(\mathcal{H}^-)$ for more general accelerating, rotating and charged Plebanski–Demianski black hole. We also determine the entropy product, entropy sum, sum of the angular velocities, temperature–entropy relations, black