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On a relation of the angular frequency to the Aharonov–Casher geometric phase in a quantum dot

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ABSTRACT

By analysing the behaviour of a neutral particle with permanent magnetic dipole moment confined to a quantum dot in the presence of a radial electric field, Coulomb-type and linear confining potentials, then, an Aharonov–Bohm-type effect for bound states and a dependence of the angular frequency of the system on the Aharonov–Casher geometric phase and the quantum numbers associated with the radial modes, the angular momentum and the spin are obtained. In particular, the possible values of the angular frequency and the persistent spin currents associated with the ground state are investigated in two different cases.

1. Introduction

In mesoscopic systems, the dependence of the energy levels on geometric quantum phases [1–4] is a well-known quantum effect in the literature [5–18]. A particular case is the dependence of the spectrum of energy on the Aharonov–Bohm geometric quantum [19] when the movement of an electron is restricted to two concentric cylinders [20,21], or an one-dimensional ring [22], or two-dimensional rings [15–17,23] without interacting with the magnetic field. This particular case is called as the Aharonov–Bohm effect for bound states [24]. Further, analogues of the Aharonov–Bohm effects for bound states have been investigated in mesoscopic systems [5–8] associated with the Berry phase [1] and the Aharonov–Anandan quantum phase [2]. With respect to neutral particles,

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Aharonov–Bohm-type effects for bound states have been explored in quantum rings [9–14,25,26] due to the interaction of the permanent magnetic dipole moment of the neutral particle with an electric field that gives rise to the Aharonov–Casher effect [27].

The objective of this work is to investigate Aharonov–Bohm-type effects for bound states [24] that arise in a two-dimensional quantum dot [15–17] due to the interaction of the permanent magnetic dipole moment of a neutral particle with a radial electric field under the influence of scalar potentials. By searching for bound state solutions, we show that the energy levels depend on the Aharonov–Casher geometric quantum phase [27] and there exists a restriction of the values of the angular frequency of the system, where the possible values of this angular frequency depend on the quantum numbers associated with the radial modes, the angular momentum and the spin, and the Aharonov–Casher geometric phase [27]. Due to this dependence of the energy levels on the geometric quantum phase, we show that persistent spin currents [9–14,18] can arise in the two-dimensional quantum dot. A particular contribution to the persistent spin currents in the quantum dot comes from the dependence of the angular frequency on the geometric quantum phase.

The structure of this paper is as follows: in Section 2, we investigate an Aharonov–Bohm-type effect for bound states [24] or an Aharonov–Casher effect for bound states [26] that arises in a two-dimensional quantum dot [15–17] under the influence of a linear scalar potential. In particular, we obtain the persistent spin currents associated with the ground state of the system; in Section 3, we investigate an Aharonov–Casher effect for bound states [26] in a quantum dot under the influence of a Coulomb-type and a linear scalar potentials. As a particular case, we show that the possible values of the angular frequency associated with the ground state of the system are determined by a third degree algebraic equation in order that bound state solutions can be obtained; in Section 4, we present our conclusions.

2. Influence of a linear scalar potential

In this section, we investigate quantum effects on a neutral particle with a permanent magnetic dipole moment confined to a quantum dot under the influence of a linear scalar potential when the magnetic dipole moment of the neutral particle interacts with an electric field. As shown in Refs. [25,27–31], the quantum dynamics of a neutral particle with a permanent magnetic dipole moment that interacts with electric and magnetic fields is described by (with $\hbar = c = 1$):

$$i\frac{\partial\psi}{\partial t} = \frac{\hat{\pi}^2}{2m}\psi - \frac{\mu^2 E^2}{2m}\,\psi + \frac{\mu}{2m}\left(\vec{\nabla}\cdot\vec{E}\right)\psi + \mu\,\vec{\sigma}\cdot\vec{B}\,\psi + V\,\psi, \tag{1}$$

where σ^i are the Pauli matrices that satisfy the relation $\left(\sigma^i\,\sigma^j + \sigma^j\,\sigma^i\right) = 2\,\eta^{ij}$, $\eta^{ij} = \text{diag}(+\,+\,+)$ and V is a scalar potential. Besides, the operator $\hat{\pi}$ is defined as

$$\hat{\pi}_k = -i\partial_k - \frac{1}{2\rho}\,\sigma^3\,\delta_{\varphi\,k} + \mu\,\left(\vec{\sigma}\times\vec{E}\right)_k. \tag{2}$$

The second term of the right-hand-side of Eq. (2) stems from the curvilinear coordinates system (cylindrical coordinates) adopted [25,31,32]. In quantum field theory in curved spacetime, this term stems from the spinorial connection [25,33].

Now, let us introduce the Aharonov–Casher effect [27]. In 1984, Aharonov and Casher [27] showed a particular case of the interaction between the permanent magnetic dipole moment and external fields, where the magnetic moment of the neutral particle interacts with an electric field produced by a linear distribution of electric charges, that is, $\vec{E} = \frac{\lambda}{\rho}\,\hat{\rho}$, where $\rho = \sqrt{x^2 + y^2}$, $\hat{\rho}$ is a unit vector in the radial direction and λ is a constant associated with the linear charge distribution along the z-axis; thus, Aharonov and Casher [27] proposed an experiment of interferometry for neutral particles in the presence of the electric field $\vec{E} = \frac{\lambda}{\rho}\,\hat{\rho}$ and showed that the wave function of the neutral particle acquires a geometric quantum phase given by

$$\phi_{\text{AC}} = \mu\oint\left(\vec{\sigma}\times\vec{E}\right)\cdot d\vec{r} = \pm 2\pi\,\mu\lambda, \tag{3}$$