Annals of Physics 372 (2016) 523–543

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Quantum decoration transformation for spin models

F.F. Braz, F.C. Rodrigues, S.M. de Souza, Onofre Rojas *

Departamento de Física, Universidade Federal de Lavras, CP 3037, 37200-000, Lavras-MG, Brazil

ARTICLE INFO

Article history:
Received 6 March 2016
Accepted 13 July 2016
Available online 25 July 2016

Keywords:
Quantum decoration transformation
Heisenberg model
Quantum spin models

ABSTRACT

It is quite relevant the extension of decoration transformation for quantum spin models since most of the real materials could be well described by Heisenberg type models. Here we propose an exact quantum decoration transformation and also showing interesting properties such as the persistence of symmetry and the symmetry breaking during this transformation. Although the proposed transformation, in principle, cannot be used to map exactly a quantum spin lattice model into another quantum spin lattice model, since the operators are non-commutative. However, it is possible the mapping in the "classical" limit, establishing an equivalence between both quantum spin lattice models. To study the validity of this approach for quantum spin lattice model, we use the Zassenhaus formula, and we verify how the correction could influence the decoration transformation. But this correction could be useless to improve the quantum decoration transformation because it involves the second-nearest-neighbor and further nearest neighbor couplings, which leads into a cumbersome task to establish the equivalence between both lattice models. This correction also gives us valuable information about its contribution, for most of the Heisenberg type models, this correction could be irrelevant at least up to the third order term of Zassenhaus formula. This transformation is applied to a finite size Heisenberg chain, comparing with the exact numerical results, our result is consistent for weak xy-anisotropy coupling. We also apply to bond-alternating

* Corresponding author.
E-mail address: ors@dfi.ufla.br (O. Rojas).

http://dx.doi.org/10.1016/j.aop.2016.07.007

Ising–Heisenberg chain model, obtaining an accurate result in the limit of the quasi-Ising chain.

1. Introduction

A considerable number of classical decorated Ising models have been solved using the decoration transformation proposed in the 1950s by M.E. Fisher [1] and Syozi [2], since that, this transformation was useful to study decorated Ising lattice in triangular, honeycomb, Kagomé, and bathroom-tile lattices [3–6], as well as the Union Jack (centered square) [7] and the square Kagomé [8] lattice, later pentagonal lattice also was considered by Urumov [9] and by Rojas et al. [10] among others. Motivated by the above results, later this approach was generalized in Ref. [11] for arbitrary spin, such as the classical–quantum spin models. The decoration transformation can also be applied to classical–quantum spin models, such as Ising–Heisenberg models. Several quasi-one-dimensional models have been investigated, similar to that diamond-like chain in [12–20] and references therein, as well as two-dimensional lattice spin models [21–29]. Furthermore, it can be applied even for three-dimensional decorated systems [30], this approach can also be applied combining with Monte Carlo simulation for 3D systems [31,32].

Classical decoration transformation could be applied beyond spin models, such as localized Ising spins and itinerant electrons in two-dimensional models as discussed by Strecka et al. [23]. Later, the decoration transformation approach has also been applied to spinless interacting particles, which shows the possibility of application for interacting electron models [33]. Due to these meaningful signs of progress, Strecka [34] discussed this transformation in a more detailed fashion, following the approach proposed in Ref. [11] for the case of quantum–classical models. Recently, another interesting transformation [35] was also suggested to avoid applying several steps of decoration transformations, by using just one transformation. Alternatively, Derzhko et al. [36] proposed a perturbative approach to study the almost Ising–Heisenberg diamond chain, by adding a small contribution in XY part.

It is of great importance the extension of classical decoration transformation for the quantum spin models, because most of the real materials could be well described by Heisenberg type models. Besides, recent investigations concerning thermal entanglement have motivated also this mapping such as q-bits bonded by Heisenberg coupling with finite number of sites. Thus, quantum decoration transformation could be potentially applied for small quantum systems in [37–40] and references therein.

In this paper, we present a pure quantum decoration transformation for a quantum mixed or decorated quantum spin model into an effective quantum spin model. The main difference between the classical and quantum transformation is the non-commutative property; consequently, the Boltzmann factor becomes an operator. A basic idea of quantum decoration transformation already has been discussed for a particular case of diluted Heisenberg model [41]. To introduce a quantum version of decoration transformation for Heisenberg spin models into a uniform spin-1/2 Heisenberg model, we will follow the basic idea used by Dunn and Essam [41], as well as by M.E. Fisher [1] and Syozi [2].

This paper is organized as follows, in Section 2 we present the two-leg quantum decoration transformation, where is included a couple of applications. In Section 3 we show the star-triangle decoration transformation, we also give a couple of applications for the star-triangle transformation. Whereas, in Section 4 we discuss how to apply for a quantum spin lattice model, the correction of the transformation can be obtained using the Zassenhaus formula. Besides, we apply for finite size Heisenberg model as well as for bond alternating Ising–Heisenberg model. Finally, in Section 5 we give our conclusion and perspectives.