Annals of Physics 372 (2016) 553–563

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Hysteresis in a superfluid atom circuit

Tieling Song, D.L. Zhou *

Beijing National Laboratory for Condensed Matter Physics, and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

ARTICLE INFO

Article history:
Received 2 February 2016
Accepted 13 July 2016
Available online 22 July 2016

Keywords:
Hysteresis
BEC
Dissipation

ABSTRACT

We study a hysteresis phenomenon in a rotating BEC with a weak link in a quasi-one-dimensional torus by proposing a microscopic theoretical model including a dissipation bath. By analyzing the role of dissipation and the decay rates of all the energy levels, we are able to give a microscopic interpretation of hysteresis recently observed in the experiment and confirm that the hysteresis is the result of the presence of metastable state. In particular, we obtain the hysteresis loops in a quench process just as that in the experiment. We also find that the shape and size of the hysteresis loop change drastically with the strength of the link.

1. Introduction

Atomtronics [1–3] is an emerging interdisciplinary field that focuses on ultracold atom analogs of electronic circuits and devices. A series of theoretical demonstrations [4–7] and impressive experiments [8,9] with BECs, a testing bed for atomtronics, have established this analogy, especially for Josephson effects [10–12], Bloch Oscillations [13]. Nevertheless, hysteresis in a superfluid atomtronic gas, which is considered to be essential to the realization of an atomic-gas superconducting quantum interference device (SQUID), has not been directly observed until recently [14]. In this experiment, both hysteresis and the quantization of flow have been observed in an atomtronic circuit formed from a ring of superfluid BEC obstructed by a rotating weak link. Just as the essential role it plays in the electronic circuit, the realization of hysteresis in atomtronic circuit will greatly accelerate the development of atomtronics because the controllability of hysteresis is crucial for the requirements of practical devices.

* Corresponding author.
E-mail address: zhoudl72@iphy.ac.cn (D.L. Zhou).

http://dx.doi.org/10.1016/j.aop.2016.07.009

Hysteresis, widely used in electronic circuits, is the phenomenon where the state of a physical system depends upon its history. The canonical example of hysteresis in a classical system is that in ferromagnetism. According to the Landau theory of phase transitions [15]: the energy landscape, which changes with the applied magnetic field, is produced by calculating the energy of the system as a function of magnetization. Hysteresis occurs when the energy landscape has two local minima separated by an energy barrier. At some critical field, the barrier disappears and the system has only a global minimum.

At a rudimentary level, hysteresis manifests itself as the competition between experimental time scales and internal time scales, the former are determined by the frequencies of the applied perturbation and the latter are governed by relaxation, decay and so on [16]. To observe a hysteresis in a quantum system, we require that some eigenstates of the system are metastable, namely, we can safely neglect the decays of those states in the experimental time scale. In this paper, we investigate the essential role that the metastable states play in the formation of hysteresis.

The swallowtail energy loop is a generic feature of hysteresis in an atomtronic circuit [17–20]. However, much of the study of swallowtails is rooted in the exact solutions to the Gross–Pitaevskii equation [21,22]. Recently, the dynamics simulation in a toroidal BEC [23,24] is also confined to the mean-field approximation.

In our study we introduce a microscopic model for the dissipation, and thus we can associate the swallowtail energy loop with the existence of metastable states. By this microscopic model, we obtain the relative decay rates of all many-body states, which decide whether a metastable state exists in our system or not, and lead to a quantitative calculation of the hysteresis in our system. Our calculations show that the interaction tends to increase the size of the hysteresis loop, while the strength of the link tends to decrease it, and confirm that there exists a metastable state which results in the hysteresis loops.

2. Theoretical model

2.1. Two-mode approximation

We consider a quasi-one-dimensional dilute gas, containing N bosonic atoms in a thin annulus of radius R and cross-sectional radius $r_0 \ll R$, which rotates at frequency Ω driven by a rotating repulsive potential V [25]. The Hamiltonian of this system $\mathscr{S}$ in the rotating frame with frequency Ω is given by

$$H_S = \sum_i \left[\frac{L_{zi}^2}{2MR^2} + V(\vec{r}_i) \right] + \frac{1}{2} \sum_{ij} g\delta(\vec{r}_i - \vec{r}_j) - \Omega \sum_i L_{zi}, \tag{1}$$

where M is the atomic mass, g is the strength of contact interaction, $L_z = -i\hbar\partial/\partial\theta$ is the angular momentum operator, and the potential takes the form

$$V = \begin{cases} V_0, & |\theta| \le \theta_0, \\ 0, & |\theta| > \theta_0, \end{cases} \tag{2}$$

which depletes the density in a small portion of the ring and thereby creates a weak link.

In terms of single-particle eigenstates $\psi_l(\theta) = e^{il\theta}/\sqrt{2\pi}$ of L_z with angular momentum $l\hbar$, the Hamiltonian can be written as

$$H_S = \frac{1}{2}\hbar\Omega_0 \sum_j (j - \bar{\Omega})^2 a_j^\dagger a_j + \sum_{j \ne k} \frac{V_0 \sin(j-k)\theta_0}{(j-k)\pi} a_j^\dagger a_k + \frac{1}{2} g \sum_{jkm} a_j^\dagger a_k^\dagger a_{k-m} a_{j+m}, \tag{3}$$

where $\Omega_0 = \hbar/MR^2$, $\bar{\Omega} = \Omega/\Omega_0$, and $a_j^\dagger$ (a_j) is the creation (annihilation) operator of boson with angular momentum $j\hbar$.

In the experiment [14], a two-step sequence is used to observe hysteresis in a BEC of ^{23}Na atoms. The BEC is firstly prepared in either the $n = 0$ or the $n = 1$ circulation state by either not rotating the weak link or by rotating it at Ω_1. Then the weak link is rotated at various angular velocities Ω_2 for a