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Perturbative vacuum wavefunctional for gauge theories in the Milne space

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ABSTRACT

The spectrum of vacuum fluctuations in the Milne space (i.e. the $\tau - \eta$ coordinate system) is an important ingredient in the thermalization studies in relativistic heavy ion collisions. In this paper, the Schrödinger functional for the gauge theory perturbative vacuum is derived for the Milne space. The Wigner-transform of the corresponding vacuum density functional is also found together with the propagators. We finally identify the fluctuation spectrum in vacuum, and show the equivalence between the present approach and the symplectic product based method (Dusling et al., 2011; Epelbaum and Gelis, 2013).

1. Introduction

When two heavy ions collide at ultra-relativistic energies, the collision produces a system that will eventually evolve into quark–gluon plasma. In the CGC-Glasma picture of the initial stage [1–11], each nucleus is initially composed of two parts; a thin disk made up of large x partons acting as a static color source and the strong static gluon field generated by the disk. Gluon quanta in this gluon field are the small x partons in the nuclear parton distribution. When the nuclei collide, the large x partons mostly pass through each other almost unscathed (except occasionally producing hard jets) but the gluon fields from each nuclei start to interact strongly with each other.

In the lab frame, the two Lorentz contracted nuclei start to overlap at $t \approx z \approx 0$. At any $t > 0$, the system is then composed of the two disks located at $z \approx \pm t$ and the gluon field stretched between them. In a very simplified picture, one could think of the gluonic system as a uniform 'string' that

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is being stretched between two ends pulling away with the speed of light. In this 'string' picture, a segment of string at position z at time t will have the speed $v = z/t$. Since the length of the string is linearly proportional to time t, this speed will remain constant for any given segment of the string. This segment therefore can be uniquely labeled by the space–time rapidity

$$\eta = \tanh^{-1}(z/t) \tag{1}$$

and its local time is the proper time

$$\tau = t/\gamma = \sqrt{t^2 - z^2} \tag{2}$$

which are nothing but the Milne coordinates. Therefore, this coordinate system is the most natural one to describe the evolution of such systems. Note that at $\tau = 0$ the spacetime rapidity η is not well defined. Accordingly, theory of the created matter must be formulated in this space with the restriction that τ is strictly non-zero and positive definite.

The usual perturbation theory in Minkowski space is concerned about calculating the scattering amplitude $\mathcal{M} = \langle \text{out}|\hat{T}|\text{in}\rangle$ where $\hat{T}$ is the scattering operator, the $|\text{in}\rangle$ state is defined at the remote past $t = -\infty$, and the $|\text{out}\rangle$ state is defined at the remote future $t = +\infty$. Hence, the problem being solved is not an initial value problem but a boundary value problem. In contrast, the quantum field theory in the Milne space is most naturally formulated as an initial value problem since τ is restricted to be positive, and also $\tau = 0^+$ (the trajectory of the color sources) represents an actual physical boundary in any ultra-relativistic heavy ion collisions.

The initial value problem in quantum mechanics is most naturally formulated in terms of the expectation value $\langle \hat{O}(\tau)\rangle = \langle \text{in}|U^\dagger(\tau)\hat{O}U(\tau)|\text{in}\rangle$ where $\hat{U}(\tau)$ is the time evolution operator. Computation of the expectation value requires the path integral on the closed time path (CPT) [12,13].[1] Explicitly, using the scalar field theory as an example,

$$\langle \hat{O}(\tau)\rangle = \int [d\phi^f] \int^{\phi^f} \mathcal{D}\phi_1 \int^{\phi^f} \mathcal{D}\phi_2\, \rho_0[\phi_1^0, \phi_2^0]\, e^{i\int_{\tau_0}^{\tau_f} d\tau\, \int d^3\tilde{x}\,(\mathcal{L}(\phi_1)-\mathcal{L}(\phi_2))}\, O[\phi_1, \phi_2] \tag{3}$$

where

$$\rho_0[\phi_1^0, \phi_2^0] = \left\langle \phi_1^0 \middle| \text{in}\right\rangle \left\langle \text{in} \middle| \phi_2^0\right\rangle \tag{4}$$

is the matrix element of the initial density operator $\hat{\rho}_0 = |\text{in}\rangle\langle\text{in}|$. The details of $O[\phi_1, \phi_2]$ depend on the time-ordering structure of the operators in $\hat{O}$. The field ϕ_1 lives on the forward going time line and the field ϕ_2 lives on the backward going time line. Here $\tau_0 > 0$ is the initial time, and all the quantities with a 0 subscript are evaluated at τ_0. The two fields $\phi_{1,2}$ share the same boundary value ϕ_f at the final time which is then traced. Here, the functional integral measure $\mathcal{D}\phi$ represents integrating over a function of both space and time and $[d\phi^f]$ represents integrating over a function of space at a fixed time (in this case $t = t_f$).

Throughout this paper, we denote the position and the position integral in the (longitudinal) Milne space with a tilde. That is, $\tilde{x} = (\mathbf{x}_\perp, \eta)$ and $d^3\tilde{x} = d^2x_\perp d\eta$. The subscript $\perp$ denotes only the x and y components of a vector quantity.

To compute $\langle \hat{O}(\tau)\rangle$ in Eq. (3), it is crucial to have the initial wave-functional $\left\langle \phi_1^0 \middle| \text{in}\right\rangle$, equivalently the matrix element $\rho_0[\phi_1, \phi_2]$. The goal of this paper is to derive the initial wave-functional for the perturbative Abelian gauge theory vacuum in the Milne space. An initial attempt in this direction was made in [14]. To convince oneself that the Schrödinger functional approach is the right way to tackle our problem, one can look at sections 2.1 and 2.2 of the same reference [14], where the simple example of an harmonic oscillator was treated with both direct computation and the Schrödinger functional method, leading to equivalent results. Within perturbation theory, non-Abelian gauge theory vacuum may be obtained by having $N_c^2 - 1$ copies of the Abelian vacuum. We also verify that the propagators

[1] This formalism is variously known as the in–in formalism, Schwinger–Keldysh formalism, and also Keldysh–Schwinger formalism.