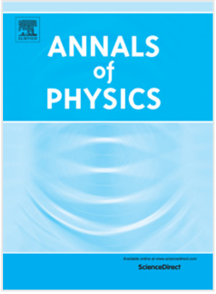

Quantum particle confined to a thin-layer volume: Non-uniform convergence toward the curved surface

Yong-Long Wang, Hong-Shi Zong

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Quantum particle confined to a thin-layer volume: Non-uniform convergence toward the curved surface

Yong-Long Wang[1,2,*] and Hong-Shi Zong[1,3,4,†]
[1] Department of Physics, Nanjing University, Nanjing 210093, China
[2] Department of Physics, School of Science, Linyi University, Linyi 276005, China
[3] Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing 210093, China and
[4] State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing 100190, China

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We clearly refine the fundamental framework of the thin-layer quantization procedure, and further develop the procedure by taking the proper terms of degree one in q_3 (q_3 denotes the curvilinear coordinate variable perpendicular to curved surface) back into the surface quantum equation. The well-known geometric potential and kinetic term are modified by the surface thickness. Applying the developed formalism to a toroidal system obtains the modification for the kinetic term and the modified geometric potential including the influence of the surface thickness.

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I. INTRODUCTION

The thin-layer quantization formalism was first introduced in 1971 by H. Jensen and H. Koppe [1], and generalized by R. C. T. da Costa (JKC) to investigate the quantum dynamics for a constrained single particle [2] and for constrained multiple particles [3]. In the three original papers, the fundamental framework of the JKC procedure was actually employed, but it was not explicitly defined.

With the development of the theoretical condensed matter physics, two dimensional (2D) curved systems are extensively investigated to study new physical effects that depend on both the curvature and the electromagnetic field, such as Aharonov-Bohm effect [4–6], quantum Hall effect [7, 8]. Recently, some experiments were designed to investigate the geometric effects on the transport in photonic topological crystals [9], on the proximity effects [10] and on the electron states [11]. Both the theoretical and experimental developments have attracted tremendous interest in the generalization of the JKC procedure to discuss a curved system with an electromagnetic field [12–14, 16, 17]. Under certain conditions, for the electromagnetic field a proper gauge should be chosen [13]. At the same time, the presence of the electromagnetic field determines that the motion equation of the vector potential for the electromagnetic field should be included [14]. However, there is no an explicit fundamental framework of the JKC procedure to study the quantum equation, the chosen gauge and the motion of the vector potential simultaneously. The absence may lead to some calculational ambiguities [15]. Generally, the curved system in an electromagnetic field can be described by a canonical action integral [14, 18]. By performing partial integration, the action can be divided into a volume integral and a surface integral. By varying the volume integral, the mentioned quantum equation can be obtained. In the general form, the absence of the fundamental framework of the JKC procedure maybe lead some calculational ambiguities for the simplifications of the integrals.

In the present paper, we will explicitly refine the fundamental framework of the JKC procedure. The procedure determines that the limit $q_3 \to 0$ (q_3 is the curvilinear coordinate variable perpendicular to the curved surface) must be performed after calculating all derivatives with respect to q_3, and the limit $d \to 0$ (d denotes the thickness of the curved surface) must be done after integrating all integrations of q_3. Employing the framework, we reconsider a spin-less charged particle confined on a curved surface in an electromagnetic field. For the considered system, the Coulomb gauge, which is chosen for the vector potential of the electromagnetic field, the motion of the vector potential, which couples to the three-dimensional (3D) electric currents $\vec{J}$, and the Schrödinger equation are together originally defined in (3D) curved space. It is more physical and actual that a curved system has a certain thickness [19]. We develop the fundamental framework by taking the suitable terms of degree one in q_3 back into the surface quantum dynamics. These terms modify the well-known geometric potential and kinetic term. These modifications can approximately describe the effects of the surface thickness. As an example for the applications of the developed procedure, we consider a spin-less charged particle constrained in a thin toroidal volume in the presence of an electromagnetic field.

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[*] Electronic address: wangyonglong@lyu.edu.cn
[†] Electronic address: zonghs@nju.edu.cn