



A new class of asymptotically non-chaotic vacuum singularities^{☆,☆☆}



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ABSTRACT

The BKL conjecture, stated in the 1960s and early 1970s by Belinski, Khalatnikov and Lifschitz, proposes a detailed description of the generic asymptotic dynamics of spacetimes as they approach a spacelike singularity. It predicts complicated chaotic behaviour in the generic case, but simpler non-chaotic one in cases with symmetry assumptions or certain kinds of matter fields.

Here we construct a new class of four-dimensional vacuum spacetimes containing spacelike singularities which show non-chaotic behaviour. In contrast with previous constructions, no symmetry assumptions are made. Rather, the metric is decomposed in *Iwasawa variables* and conditions on the asymptotic evolution of some of them are imposed. The constructed solutions contain five free functions of all space coordinates, two of which are constrained by inequalities.

We investigate continuous and discrete isometries and compare the solutions to previous constructions. Finally, we give the asymptotic behaviour of the metric components and curvature.

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1. Introduction

1.1. Singularities in general relativity

When Albert Einstein presented his theory of General Relativity in 1915 he did not give any non-trivial exact solutions to its field equations. Due to the complicated non-linear structure of the equa-

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tions, he did not expect any to exist and calculated physical predictions using perturbation theory [1]. To his surprise, less than a month later, Karl Schwarzschild sent him a letter containing the Schwarzschild metric, a spherically symmetric solution of the vacuum Einstein equations. It is given, in Schwarzschild coordinates, as

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} dt^2 + \left(1 - \frac{2m}{r}\right) dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

This solution contains, in these coordinates, an apparent singularity at $r = 2m$ where the tt component of the metric diverges. This is the event horizon, which Schwarzschild set as the origin of his coordinate system. The solution also contains a real singularity at $r = 0$ which was found by David Hilbert in 1917. Hilbert considered both singularities real, as they could not be removed by an everywhere smooth and invertible coordinate transformation. In hindsight his requirement was too strict: The fact that in Schwarzschild coordinates the tt component of the metric diverges at $r = 2m$ simply means that these coordinates are badly chosen, indeed to transform from coordinates which do not show the apparent singularity to Schwarzschild coordinates requires a transformation which diverges at $r = 2m$.

In 1921 and 1922 Paul Painlevé and Allvar Gullstrand independently discovered a spherically symmetric vacuum solution containing only a single singularity at $r = 0$ [2,3]. It was not realized at the time that this solution can be obtained from the Schwarzschild one by a coordinate transformation, i.e. it describes the same physical spacetime. This was finally discovered by Georges Lemaître in 1932, who also correctly identified the $r = 2m$ singularity as an apparent singularity caused by the choice of coordinates [4]. The singularity at $r = 0$ cannot be removed by a coordinate transformation as the Kretschmann scalar, given by $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, diverges there. This is a scalar quantity, constructed by contracting all indices of the Riemann tensor with itself, and is therefore independent of the chosen coordinate system.

Despite this advance, the status of real singularities, such as the one appearing in the Schwarzschild or the cosmological FLRW solutions, was unclear. It was widely believed that they were an artefact of the symmetry assumptions made to obtain explicit solutions and had no relevance for the real world [5]. The idea was that, similarly to the Newtonian case, if matter was not perfectly symmetrically rushing towards a central point, the resulting angular momentum would prevent the formation of a singularity.

The singularity theorems of Penrose and Hawking [6,7] proved the opposite. They state that, given a trapped surface, an energy condition, and an assumption on the global structure of spacetime (e.g. no closed timelike curves), a singularity, in the sense of geodesic incompleteness, has to form. As small perturbations of an explicit solution containing a singularity would preserve the trapped surface, the perturbed solution also contains a singularity. These theorems, however, do not give any information about the nature of the predicted singularities, or about the behaviour of the metric near them. Indeed they do not even predict diverging curvature, only the existence of some geodesics, which cannot be extended beyond a finite value of the affine parameter along them.

1.2. The BKL conjecture

In a series of works, beginning in 1963, Belinski, Khalatnikov and Lifschitz (BKL) conjectured, based on heuristic arguments, that the dynamics of a generic spacetime containing a spacelike singularity would drastically simplify when the singularity is approached [8,9]. They claimed that time derivatives of the metric would dominate compared to space derivatives, causing different spatial points to effectively decouple and turning the Einstein equations into a system of ODEs at each point. The solution of these ODEs is a generalization of the Kasner metric, an explicit, homogeneous (but anisotropic) solution of the Einstein equations describing a spacetime which expands in some directions and contracts in others. It is given by

$$ds^2 = -dt^2 + \sum_{j=1}^d t^{2p_j} (dx^j)^2, \quad (1.1)$$

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