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Quantum unitary dynamics in cosmological spacetimes

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ABSTRACT

We address the question of unitary implementation of the dynamics for scalar fields in cosmological scenarios. Together with invariance under spatial isometries, the requirement of a unitary evolution singles out a rescaling of the scalar field and a unitary equivalence class of Fock representations for the associated canonical commutation relations. Moreover, this criterion provides as well a privileged quantization for the unscaled field, even though the associated dynamics is not unitarily implementable in that case. We discuss the relation between the initial data that determine the Fock representations in the rescaled and unscaled descriptions, and clarify that the S-matrix is well defined in both cases. In our discussion, we also comment on a recently proposed generalized notion of unitary implementation of the dynamics, making clear the difference with the standard unitarity criterion and showing that the two approaches are not equivalent.

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1. Introduction

In a series of works completed during the last ten years [1–6], strong results have been obtained by us and other collaborators concerning the Fock quantization of scalar fields with an effective time dependent mass and the preservation of unitarity. In particular, when applied to the quantization of free (test) fields in an expanding background spacetime with homogeneous and isotropic spatial sections [i.e., a Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime], these results allow one to select a privileged parametrization of the field variable, that involves a rescaling of the original field by means of the time dependent scale factor of the background [7,6,8].

For definiteness, let us consider the case of a flat FLRW spacetime, with compact spatial sections isomorphic to the 3-torus. Using conformal time, the metric can be written in the form

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{x}^{\,2}\right), \qquad (1)$$

where $a(\eta)$ is the scale factor and $d\vec{x}^{\,2}$ is the standard metric on the 3-torus. It is well known that the free scalar field equation

$$\left(\Box - m^2\right)\phi = 0, \qquad (2)$$

acquires its simplest form after the change of field variable

$$\chi = a\phi, \qquad (3)$$

leading to

$$\chi'' - \Delta\chi + m^2(\eta)\chi = 0, \qquad (4)$$

where $m^2(\eta) = a^2m^2 - (a''/a)$ and Δ is the standard Laplacian on the 3-torus. Besides, the box stands for the D'Alambertian of the FLRW spacetime, and the derivative with respect to the conformal time is denoted with a prime.

For the type of Eq. (4), it is known that there is a quantum Fock representation of the canonical commutation relations (CCR's) at fixed (equal) time such that the dynamics in the canonical framework is unitarily implementable. Moreover, additional research [4,7,6,8], in which we participated, has proven that this quantization is unique [among the set of Fock quantizations determined by complex structures (CS's) that are invariant under the action of spatial isometries]. It has also been shown that unitary implementation of the dynamics is impossible to achieve for the type of equations satisfied by the original unscaled field ϕ, or by means of any other rescaling different from the one introduced above: $\phi \rightarrow \chi = a\phi$.

The usefulness of the above transformation, $\phi \rightarrow \chi$, in particular to simplify the field equation, has of course been known for a long time (see e.g. [9]) and it is commonly used, even in the anisotropic cosmology context (see for instance [10]). Moreover, nothing is lost by considering the field χ, since the construction of field operators $\hat{\chi}(\eta)$ in the quantization process immediately gives rise to operators $\hat{\phi}(\eta) = a^{-1}(\eta)\hat{\chi}(\eta)$ to represent the original field. However, there is an apparent tension between the two formulations, in the light of the different properties of the dynamics in each of them. In order to try to alleviate this tension, a recent proposal of a generalized notion of unitary implementation of the dynamics has been put forward in [11], to accommodate the fact that evolution in terms of the ϕ-field description is not unitary.

Nonetheless, in our opinion this tension is somewhat artificial. The main purpose of this article is to clarify this issue, explaining how one can maintain a well-defined and non-trivial concept of evolution in the quantization, and how this concept can be employed as a selection criterion for the determination of a unique Fock quantization. The rest of the paper is organized as follows. We summarize the basic mathematical tools needed to discuss the role of the dynamics in the Fock quantization of a scalar field in Section 2. Section 3 deals with the unitary implementability of the dynamics and the determination of criteria that select a unique Fock representation for the quantization of the scalar field. Section 4 analyzes the relation between the quantization of the rescaled field and the original one, and how this rescaling affects the unitarity of the dynamics. Finally, we present a discussion of the implications of our investigations and conclude in Section 5.