Annals of Physics 363 (2015) 98–113

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Conserved momenta of a ferromagnetic soliton

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ARTICLE INFO

Article history:
Received 7 July 2015
Accepted 6 September 2015
Available online 16 September 2015

Keywords:
Conserved momentum
Ferromagnet
Soliton

ABSTRACT

Linear and angular momenta of a soliton in a ferromagnet are commonly derived through the application of Noether's theorem. We show that these quantities exhibit unphysical behavior: they depend on the choice of a gauge potential in the spin Lagrangian and can be made arbitrary. To resolve this problem, we exploit a similarity between the dynamics of a ferromagnetic soliton and that of a charged particle in a magnetic field. For the latter, canonical momentum is also gauge-dependent and thus unphysical; the physical momentum is the generator of magnetic translations, a symmetry combining physical translations with gauge transformations. We use this analogy to unambiguously define conserved momenta for ferromagnetic solitons. General considerations are illustrated on simple models of a domain wall in a ferromagnetic chain and of a vortex in a thin film.

1. Introduction

The definition of linear and angular momenta of a ferromagnet is a subject with a long history, surprising results, and a lingering controversy [1–12]. It was realized in the 1970s that the linear momentum of a ferromagnetic soliton is determined not by its velocity but rather by its configuration [1,2]. Although this sounds counterintuitive, one needs to realize that our physical intuition is based on experience with massive objects, for which an external force generates proportional acceleration. Spins in a ferromagnet behave differently: like fast-spinning gyroscopes, they precess at an angular velocity proportional to the external torque. An external force $\mathbf{F}$ acting during a short time interval dt increments the velocity of a Newtonian particle, $\mathbf{F}dt = m\,d\mathbf{v}$. For a spin, an external torque τ affects its

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http://dx.doi.org/10.1016/j.aop.2015.09.004
0003-4916/

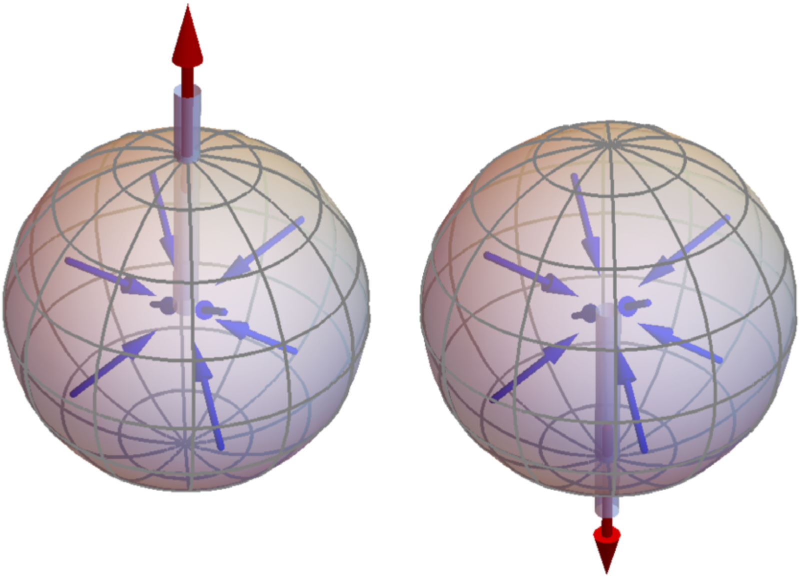

Fig. 1. Magnetic field $\mathbf{b}$ of a magnetic monopole with a net flux $-4\pi\mathcal{J}$ (blue arrows). A Dirac string at $\mathbf{m}_s$ (tube) carries an equal and opposite magnetic flux $+4\pi\mathcal{J}$ (red arrow). Standard choices are $\mathbf{m}_s = \pm\hat{\mathbf{z}}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

orientation, $\tau\, dt = d\mathbf{S}$. Thus the linear momentum of a ferromagnetic soliton is a function of its collective coordinates, rather than velocities.

The earliest derivations of linear momentum in a ferromagnet followed the above qualitative reasoning and analyzed the configurational change of a soliton under a specified external perturbation. For example, a domain wall in a ferromagnetic chain pushed by an external magnetic field increments its azimuthal angle in proportion to the impulse of the force exerted by the field. The linear momentum of the domain wall is therefore proportional to its azimuthal angle [1].

On a deeper level, momenta are conserved quantities related to global symmetries. By Noether's theorem, invariance of the Lagrangian under translations and rotations gives rise to the conservation of linear and angular momenta. The precessional dynamics of the magnetization field $\mathbf{m}(\mathbf{r}, t)$ of unit length is represented in the Lagrangian not by a kinetic energy, but rather by a Berry-phase term $\mathcal{L}_B = \mathbf{a} \cdot \partial_t \mathbf{m}$. Here $\mathbf{a}(\mathbf{m})$ is the gauge potential of a magnetic monopole [13] whose field is

$$\mathbf{b}(\mathbf{m}) = \nabla_{\mathbf{m}} \times \mathbf{a}(\mathbf{m}) = -\mathcal{J}\mathbf{m}. \tag{1}$$

$\mathcal{J}$ is spin density. Eq. (1) does not fully specify $\mathbf{a}(\mathbf{m})$: any gauge transformation

$$\mathbf{a}'(\mathbf{m}) = \mathbf{a}(\mathbf{m}) + \nabla_{\mathbf{m}}\chi(\mathbf{m}) \tag{2}$$

preserves $\mathbf{b} = \nabla_{\mathbf{m}} \times \mathbf{a}$. It is known that the linear momentum $\mathbf{p}$ derived from Noether's theorem is gauge-dependent [8,11]. This is a cause for concern: physical quantities should be gauge-invariant.

A different but related problem arises for angular momentum $\mathbf{J}$. Although the magnetic field of a monopole (1) is spherically symmetric, its gauge potential is not. The best we can do is to make it axially symmetric [3]:

$$\mathcal{L}_B = \mathbf{a} \cdot \partial_t \mathbf{m} = \mathcal{J}\frac{\mathbf{m}_s \times \mathbf{m}}{1 - \mathbf{m}_s \cdot \mathbf{m}} \cdot \partial_t \mathbf{m}. \tag{3}$$

The singular direction $\mathbf{m}_s$ is the location of a Dirac string carrying away the magnetic flux $4\pi\mathcal{J}$, Fig. 1. The axial symmetry of the Berry-phase term in the Lagrangian limits us to just one conserved