Annals of Physics 364 (2016) 261–273

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Particle detection and non-detection in a quantum time of arrival measurement

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HIGHLIGHTS

- The time-evolved position density is contained in the standard TOA distribution.
- Particle may quantum mechanically arrive at a given point without being detected.
- The eigenstates of the standard TOA operator are linked to the two-slit experiment.

ARTICLE INFO

Article history:
Received 21 July 2015
Accepted 17 November 2015
Available online 2 December 2015

Keywords:
Quantum mechanics
Quantum time of arrival
Wave function collapse
Two-slit experiment

ABSTRACT

The standard time-of-arrival distribution cannot reproduce both the temporal and the spatial profile of the modulus squared of the time-evolved wave function for an arbitrary initial state. In particular, the time-of-arrival distribution gives a non-vanishing probability even if the wave function is zero at a given point for all values of time. This poses a problem in the standard formulation of quantum mechanics where one quantizes a classical observable and uses its spectral resolution to calculate the corresponding distribution. In this work, we show that the modulus squared of the time-evolved wave function is in fact contained in one of the degenerate eigenfunctions of the quantized time-of-arrival operator. This generalizes our understanding of quantum arrival phenomenon where particle detection is not a necessary requirement, thereby providing a direct link between time-of-arrival quantization and the outcomes of the two-slit experiment.

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http://dx.doi.org/10.1016/j.aop.2015.11.008
0003-4916/

1. Introduction

Time of arrival (TOA) has always been associated with particle appearance [1]. This is the reason why attempts to construct a quantum TOA distribution use particle detection as its primary assumption [2–7]. Now, the probability of finding a particle in the neighborhood of X at an instant of time t is already encapsulated in the position density distribution $|\psi(X,t)|^2$. If particle detection is indeed necessary for quantum arrival, then the TOA distribution should resemble both the temporal and spatial profiles of the time-evolved position density. Following the standard formulation of quantum mechanics, a TOA operator can be obtained by quantizing the classical TOA. The spectral resolution of the quantum TOA operator can then be used to generate the TOA distribution for a given initial state. Such distribution is often referred to as the standard TOA distribution [8,9]. However, the standard TOA distribution fails to give the correct temporal and spatial profiles of the time-evolved position density for an arbitrary initial state. This is due to the suppression of the interference arising from the positive and negative momentum components of the initial state [10,11]. We are now confronted with the following problems: Is the standard formulation (i.e. quantization of classical observable followed by spectral analysis) wrong? If not, then what does the quantized TOA operator actually measure? Is it still possible to consistently extract the time-evolved position distribution from the standard TOA distribution? This paper will address these questions.

In this work, we will show that the position distribution is indeed contained in the standard quantum TOA distribution and that the quantum arrival has a broader meaning than the classical particle detection phenomenon. We will first examine a specific case when the standard TOA fails to give the correct spatial and temporal profile of the time-evolved position distribution. Using the properties of the confined time-of-arrival (CTOA) operator, we were able to isolate the term in the unconfined standard TOA distribution that corresponds to particle appearance. In the end, we will learn that the quantization of the classical TOA gives rise to a quantum phenomenon which is at the very core of quantum mechanics, i.e. the two-slit experiment.

In [12], one of us proposed that particle appearance is a consequence of TOA measurement. That is, the initial wave function of a particle collapses into one of the confined time of arrival (CTOA) operator eigenfunctions right after the preparation and evolves according to the Schrödinger equation [13,14]. At the time equal to the CTOA eigenvalue, the evolving eigenfunction becomes a function with a singular support at the arrival point. The instant when the wave function has its minimum position uncertainty is interpreted as particle appearance in [12]. The proposed mechanism suggests that a quantum particle may be materialized via smooth unitary evolution in contrast to the abrupt collapse upon measurement. One can therefore infer a strong connection between the time-evolved position density and the TOA distributions. However, such connection was established only in the theory of CTOA operator, which is still problematic in terms of operational interpretation [15]. In this paper, we intend to reconcile the time-evolved position and the more established standard TOA distributions in the fundamental level, i.e. without any reference to a detector. We will do this by exploiting the known properties of the CTOA operator such as the two distinct unitary arrivals.

This work is organized as follows. First, we show that the particle appearance interpretation can be extended to the unconfined case in Section 2. In Section 3, we compare the TOA distribution with the time-evolved position density for different initial states. This is followed by describing the relevant features of the CTOA theory in Section 4 and using those features to extend the interpretation of the standard TOA in Section 5. Then, we relate the reinterpreted TOA measurement with the two-slit experiment in Section 6. Finally, we give our conclusion in Section 8.

2. The standard TOA distribution

A TOA operator can be constructed for a non interacting case such that it satisfies the required canonical commutation relation with the free Hamiltonian. The standard TOA operator is given by $\hat{T} = -\mu[(\hat{q}-X)\hat{p}^{-1} + \hat{p}^{-1}(\hat{q}-X)]/2$, where X is the arrival point, μ is the mass of the particle, $\hat{q}$ is the position operator and $\hat{p}$ is the momentum operator [16]. The operator $\hat{T}$, though not self adjoint, is maximally symmetric and therefore can still be used to generate the desired distribution.