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Reformulations of Yang–Mills theories with space–time tensor fields

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ABSTRACT

We provide the reformulations of Yang–Mills theories in terms of gauge invariant metric-like variables in three and four dimensions. The reformulations are used to analyze the dimension two gluon condensate and give gauge invariant descriptions of gluon polarization. In three dimensions, we obtain a non-zero dimension two gluon condensate by one loop computation, whose value is similar to the square of photon mass in the Schwinger model. In four dimensions, we obtain a Lagrangian with the dual property, which shares the similar but different property with the dual superconductor scenario. We also make discussions on the effectiveness of one loop approximation.

1. Introduction

Regarding quantum Yang–Mills theories as highly nonlinear theories, it is difficult to achieve an understanding of its infrared region by the perturbative method. There are several approaches which suggest that the infrared region could be described through reformulating the Yang–Mills fields in terms of new variables. In [1], it was proposed that the infrared limit of the $SU(2)$ Yang–Mills theory in 4 dimensions could be given by a nonlinear sigma model by using partially dual variables,[1] which are the decomposition of $SU(2)$ gauge field in terms of the coset variables of its $U(1)$ subgroup [6–8]. In [9], it was further proposed that a complete off-shell decomposition of $SU(2)$ field

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[1] For its application to the confinement problem, see [2–4] and the review article [5].

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can be implemented through the view of spin–charge separation inspired by the strong correlated electron system.

There are also proposals to reformulate Yang–Mills theories by making use of field strength variables or gauge invariant metric-like variables [10]. Similar to the $U(1)$ Maxwell theory, which can be expressed by the field strength variables, the $SU(2)$ Yang–Mills theory can also be expressed by the field strength variables albeit in terms of an infinite series [11]. In [12], the frame-like fields are used as the pre-potential, and the $SU(2)$ Yang–Mills theory is recast into a R^2 gravity theory in 3 dimensions. In [13], the authors employed the metric-like fields and proposed that Yang–Mills theories could be regarded as the diffeomorphism invariant gravity theory broken by the background dependent ether term [14].

From another different angle, the reformulation or decomposition of gauge field is useful to address physical issues which are closely related to the gauge invariance. In [15], the transverse part of gauge field is used to analyze the gauge invariant dimension two condensate [16–18], which is also been discussed by using the "remaining" part after subtracting an "Abelian" part from the original gauge field [19]. In [20], it was proposed that the gauge invariant contribution of gluon polarization to the nucleon spin can be described by decomposing the gauge field into its physical part and its pure gauge part. In [21], many properties of quantum chromodynamics are discussed by dressed gluon fields, which are gluon fields with its pure gauge parts subtracted as background fields.

Inspired by the above investigations, we attempt to provide analysis on the infrared region of Yang–Mills theory through a novel reformulation and decomposition of the gauge field. At first, we decompose the gauge field into two parts as $A^a_\mu = B^a_\mu + e^a_\mu$. Here e^a_μ can be regarded as the frame-like fields in gravity theory, which have the same transformation properties as the gauge field strength. B^a_μ can be regarded as the gauge connection in gravity theory, which can be solved in terms of e^a_μ through imposing compatibility conditions, then a reformulation of A^a_μ in terms of e^a_μ is obtained. The Yang–Mills Lagrangian can then be reformulated by using the metric variable $g_{\mu\nu}$. We show that $g_{\mu\nu}$ provides facilities to analyze the gauge invariant dimension two condensate and give gauge invariant descriptions of gluon polarization.

This paper is organized as follows. In Section 2, we discuss the $SU(2)$ Yang–Mills theory in three dimensions (3D), where a dimension two condensate with the value $\frac{\lambda^2}{\pi^4}\kappa^4$ is derived by considering the one loop quantum correction. Here κ^2 is the coupling constant in 3D, which has the mass dimension; And λ is a numerical constant produced by the one loop correction. We also provide arguments to demonstrate that the higher loop expansions are actually strong coupling expansions in Section 2.4. In Section 3, we discuss the $SU(2)$ Yang–Mills theory in four dimensions (4D). We first derive the scalar–vector sector of the full Lagrangian in Section 3.3. By means of this scalar–vector sector, we propose a duality property between a gauge invariant order parameter and a nonzero dimension two condensate. This phenomenon is similar to but different from the dual superconductor scenario in [1,9]. In Section 3.4, we give the one-loop result of the dimension two condensate in 4D, and the effectiveness of the one-loop computation is also discussed. Two Sections 2.2 and 3.2 are used to discuss the relations about the partition function between reformulated theories and Yang–Mills theories. We provide conclusions in Section 4. Several appendices are used to provide more details of the paper.

2. *SU*(2) Yang–Mills theory in three dimensions

2.1. Formulation with space–time tensor field

In 3 dimensional space–time, the Lagrangian of $SU(2)$ Yang–Mills theory is

$$\mathscr{L} = -\frac{1}{4\kappa^2}\eta^{\alpha\mu}\eta^{\beta\nu}F^a_{\alpha\beta}F^a_{\mu\nu}, \tag{1}$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu,$$