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Automatic Fourier transform and self-Fourier beams due to parabolic potential

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ABSTRACT

We investigate the propagation of light beams including Hermite–Gauss, Bessel–Gauss and finite energy Airy beams in a linear medium with parabolic potential. Expectedly, the beams undergo oscillation during propagation, but quite unexpectedly they also perform automatic Fourier transform, that is, periodic change from the beam to its Fourier transform and back. In addition to oscillation, the finite-energy Airy beams exhibit periodic inversion during propagation. The oscillating period of parity-asymmetric beams is twice that of the parity-symmetric beams. Based on the propagation in parabolic potential, we introduce a class of optically-interesting beams that are self-Fourier beams—that is, the beams whose Fourier transforms are the beams themselves.

1. Introduction

It is well known that a light beam undergoes discrete diffraction while propagating in a waveguide array, and that such a diffraction is prohibited when the refractive index of the waveguide array

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is appropriately modulated [1,2]. The phenomenon is linear, that is, obtained without invoking nonlinearity in the paraxial wave equation [3]. Likewise, in free space or a linear bulk medium a light beam will diffract unless it belongs to the family of nondiffracting beams [4,5]—a class of linear beams that attracted a lot of attention in the past few years. A celebrated member of this class is the Airy beam [4,6–10]. In optics, ways to effectively modulate light beams have always been high on research agenda. As an effective tool, a photonic potential – a "potential" embedded in the medium's index of refraction—is often used in linear optics and extensively referenced in the literature. It comes in different forms, as exemplified by vastly different photonic crystal structures. A linear potential which affects the properties of an Airy plasmon beam and is used to control acceleration of Airy beams has been reported in [11,12]. An external longitudinally-dependent transverse potential will modulate the propagating trajectory of the light beam according to the form of the potential [13]. In Refs. [14–16] authors discussed the propagation and transformation of finite-energy Airy beams in a linear medium with a parabolic potential, in which periodic inversion, oscillation, and phase transition were reported. One should recall that in a parabolic potential, light propagation is equivalent to a fractional Fourier transform (FT) [17], the fact first noted by Mendlovic and his collaborators [18,19]. The fractional FT can be conveniently introduced through the propagation in gradient-index (GRIN) media, which can further be connected with the propagation in the parabolic potential and the automatic FT—as done in this article. By now, beam propagation in GRIN media has matured to a proper part of optics that is best surveyed from a dedicated monograph [20]. In a nonlinear medium, the strongly nonlocal nonlinearity can be cast into a parabolic-like potential [21] and consequently the corresponding modulation effects more easily investigated [22–24].

In this article, we investigate the light beam management by a parabolic potential in a linear medium, theoretically and numerically. The program presented extends the traditional Fourier optics, which deals with the free-space propagation, to the realm of propagation in a parabolic potential. Since such a potential causes harmonic oscillation of light beams, it is interesting to investigate the influence of the potential on the dynamics of useful light beams, such as Hermite–Gauss (HG), Bessel–Gauss (BG), and finite energy Airy beams (ABs). Even though there are many papers dealing with the parabolic potential and linear harmonic oscillation, we believe that results obtained here have not been reported before, to the best of our knowledge.

The organization of the article is as follows. In Section 2, we describe the problem and introduce the theoretical model of beam propagation. In Section 3, we discuss the repercussions of the model on the dynamics of the beams mentioned. In Section 4, we solve the model, obtain analytical solutions, and analyze those solutions. Based on the theoretical model, we discover a class of interesting self-Fourier beams and present them in Section 5. Section 6 concludes the paper.

2. Theoretical model

The paraxial propagation of a beam in a linear medium with an external parabolic potential, is described by the dimensionless Schrödinger equation

$$i\frac{\partial \psi}{\partial z} + \frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{2}\alpha^2 x^2 \psi = 0, \tag{1}$$

where x and z are the normalized transverse coordinate and the propagation distance, respectively, scaled by some transverse width x_0 and the corresponding Rayleigh range kx_0^2. Here, $k = 2\pi n/\lambda_0$ is the wavenumber, n the index of refraction, and λ_0 the wavelength in vacuum. Parameter α scales the width of the potential. For our purposes, the values of parameters can be taken as $x_0 = 100\,\mu$m, $n = 1.45$, and $\lambda_0 = 600$ nm [25,26].

Eq. (1) has many well-known solutions; we utilize the ones that are of interest in the paraxial beam propagation. But before selecting any solutions, we perform the Fourier transform (FT) of Eq. (1), to obtain

$$i\frac{\partial \hat{\psi}}{\partial z} + \frac{1}{2}\alpha^2\frac{\partial^2 \hat{\psi}}{\partial k^2} - \frac{1}{2}k^2 \hat{\psi} = 0, \tag{2}$$