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The Casimir effect for fields with arbitrary spin

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ABSTRACT

The Casimir force arises when a quantum field is confined between objects that apply boundary conditions to it. In a recent paper we used the two-spinor calculus to derive boundary conditions applicable to fields with arbitrary spin in the presence of perfectly reflecting surfaces. Here we use these general boundary conditions to investigate the Casimir force between two parallel perfectly reflecting plates for fields up to spin-2. We use the two-spinor calculus formalism to present a unified calculation of well-known results for spin-1/2 (Dirac) and spin-1 (Maxwell) fields. We then use our unified framework to derive new results for the spin-3/2 and spin-2 fields, which turn out to be the same as those for spin-1/2 and spin-1. This is part of a broader conclusion that there are only two different Casimir forces for perfectly reflecting plates—one associated with fermions and the other with bosons.

1. Introduction

In 1948 Casimir and Polder published a long, technically complex paper about the influence of retardation on the van der Waals force [1]. The stated goal of their work was to account for discrepancies between experiments and theory concerning colloidal suspensions of large particles [2]. However, the work took on a whole new significance when, after discussing the results with Bohr, Casimir was inspired to try and re-explain his and Polder's results using the relatively new idea that the quantised electromagnetic field undergoes vacuum fluctuations. A short time later, Casimir published his now-famous paper [3] on the force of attraction between two infinite perfectly conducting parallel plates, whose presence modifies the quantised electromagnetic vacuum field. This force came to be known as the Casimir force. The calculation in [3] reproduces the results of the much more involved calculation in [1], but is remarkable in its simplicity and elegance, while also providing one of the very few macro-

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scopic manifestations of quantum field theory. The Casimir force is extremely weak so was initially nothing more than a theoretical curiosity, but as experimental methods improved the effect became measurable, and this led to rapidly increasing attention from the 1970s onwards. Since then there has been a profusion of extensions of Casimir's original work, which look into imperfectly conducting plates and different physical geometries [4–8]. There have also been a number of experimental confirmations of the effect [9–11]. The existence of the Casimir effect is often cited in standard quantum field theory textbooks as the primary justification for the reality of vacuum fluctuations, though such interpretations carry some controversy [12].

A fluctuating vacuum is a general feature of quantum fields, of which the free Maxwell field considered in [1–12] is but one example. Fermionic fields such as that describing the electron, also undergo vacuum fluctuations, consequently one expects to find Casimir effects associated with such fields whenever they are confined in some way. Such effects were first investigated in the context of nuclear physics, within the so-called "MIT bag model" of the nucleon [13]. In the bag-model one envisages the nucleon as a collection of fermionic fields describing confined quarks. These quarks are subject to a boundary condition at the surface of the 'bag' that represents the nucleon's surface. Just as in the electromagnetic case, the bag boundary condition modifies the vacuum fluctuations of the field, which results in the appearance of a Casimir force [14–18]. This force, although very weak at a macroscopic scale, can be significant on the small length scales encountered in nuclear physics. It therefore has important consequences for the physics of the bag-model nucleon [19].

The Maxwell and Dirac fields are both spinor fields, though the former is not usually described as such. It is possible to write Maxwell's equations in a form identical to the Dirac–Weyl equation that describes massless spin-$1/2$ fermions [20]. This naturally leads one to the question as to whether it is possible to use a spinor formalism to describe the Casimir effect for the Dirac (spin-$1/2$) and Maxwell (spin-1) fields in a unified way. We have shown [21] that such a unification can be accomplished using the two-spinor calculus formalism introduced by van der Waerden [22]. Moreover, this unification naturally lends itself to a generalisation, which is applicable to confined higher-spin fields. These fields include the spin-2 field associated with the so-called graviton, which appears in linearised quantum gravity, and its supersymmetric partner the spin-$3/2$ gravitino.

In this paper we will present specific results for the Casimir force associated with the fields up to spin-2. We organise our work by noting that calculations of Casimir forces broadly follow the following three steps:

1. The statement of one or more boundary conditions governing how the considered field behaves at material surfaces. These can be mathematically convenient (examples include Dirichlet [23], Neumann [24], Robin [25] and periodic [26,27] BCs) or physically-motivated (those imposed by electromagnetism [4] or by the bag model [13] for example).
2. The determination of a set of field solutions that obey the boundary conditions specified in step 1. In the simplest cases this can be achieved by direct solution of the equations of motion. However, this step is usually non-trivial, and has resulted in the development of numerous techniques including the so-called macroscopic QED [28], worldline numerics [29], the "proximity-force approximation" [30,31], certain scattering theory based methods [32], and many more.
3. The substitution of the field solutions found in step 2 into an expression for the vacuum energy of the relevant field. Upon suitable regularisation and the dropping of any boundary-independent terms, one is left with the Casimir force for some combination of: a field (Maxwell, Dirac, etc.), a boundary condition, and a physical geometry.

We will begin in Section 2 by reviewing the generalised, physically-motivated boundary conditions presented in [21]. We then find explicit field solutions for the parallel-plate geometry, and hence accomplish steps 1 and 2 given above. In Section 3 we will carry out the final step above by computing specific values for the Casimir force associated with the massless fields up to spin-2.

2. Generalised physical boundary conditions

In this section we review our generalisation of the boundary conditions (BCs) employed in the calculation of the Casimir effect associated with the spin-$1/2$ and spin-1 fields. To do this we use the