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Dynamical stability of a many-body Kapitza pendulum

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ABSTRACT

We consider a many-body generalization of the Kapitza pendulum: the periodically-driven sine–Gordon model. We show that this interacting system is dynamically stable to periodic drives with finite frequency and amplitude. This finding is in contrast to the common belief that periodically-driven unbounded interacting systems should always tend to an absorbing infinite-temperature state. The transition to an unstable absorbing state is described by a change in the sign of the kinetic term in the Floquet Hamiltonian and controlled by the short-wavelength degrees of freedom. We investigate the stability phase diagram through an analytic high-frequency expansion, a self-consistent variational approach, and a numeric semiclassical calculation. Classical and quantum experiments are proposed to verify the validity of our results.

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1. Introduction

Motivated by advances in ultra-cold atoms [1–5], the stability of periodically-driven many-body systems is the subject of several recent studies [6–14]. According to the second law of thermodynamics, isolated equilibrium systems can only increase their energy when undergoing a cyclic process. For many-body interacting ergodic systems, it is often assumed that they will heat monotonously, asymptotically approaching an infinite-temperature state [8,9,11,15]. In contrast, for small systems such as a single two-level system (spin), thermalization is not expected to occur and periodic alternations of heating and cooling (Rabi oscillations) are predicted. A harmonic oscillator can display a transition between these two behaviors, known as "parametric resonance" [16]: depending on the amplitude and frequency of the periodic drive, the oscillation amplitude either increases indefinitely, or displays periodic oscillations. An interesting question regards how much of this rich dynamics remains when many degrees of freedom are considered.

This question was addressed for example by Russomanno et al. [6], who studied the time evolution of the transverse-field Ising (TI) model, and found that it never flows to an infinite-temperature state. This result can be rationalized by noting that the TI model is integrable and can be mapped to an ensemble of decoupled two-level systems (spin-waves with a well defined wavevector), each of which periodically oscillates in time and never equilibrates. In this sense, the findings of Refs. [11–14] on periodically-driven disordered systems subject to a local driving fall into the same category: many-body localized (MBL) systems are effectively integrable because they can be described as decoupled local degrees of freedom [17–19]. Earlier numerical studies [7,20–22] found indications of a finite stability threshold in non-integrable systems as well. These findings are in contrast to the common expectation that ergodic systems should always thermalize to an infinite temperature [6,9,11,15].

To investigate this problem in a systematic way, we consider here a many-body analog of the Kapitza pendulum: the periodically-driven sine–Gordon model. This model is well suited for analytical treatments, including high-frequency expansion, Gaussian variational approaches, and renormalization-group methods. Unlike previously-studied spin systems, the present model involves continuous fields, whose energy density is not bounded from above: its infinite-temperature ensamble is therefore characterized by an infinite energy density and is easily identified. In this paper we show the emergence of a sharp "parametric resonance", separating the absorbing (infinite temperature) from the non-absorbing (periodic) regimes. This transition survives in the thermodynamic limit and leads to a non-analytic behavior of physical observables, as a function of the driving strength and/or frequency. We conjecture that this transition corresponds to a mean-field critical point of the many-body Floquet Hamiltonian. Our finding enriches the understanding of the coherent dynamics of parametrically forced system and paves the road toward the search of unconventional dynamical behavior in closed many-body systems.

2. Review of a single Kapitza pendulum

Before entering the domain of many-body physics, we briefly review the (well understood) case of a single degree of freedom. Specifically, we consider the *classical* Hamiltonian of a periodically-driven simple pendulum, the Kapitza pendulum [23]

$$H(t) = \frac{1}{2}p^2 - g(t)\cos(\phi), \quad \text{with } g(t) = g_0 + g_1\cos(\gamma t). \tag{1}$$

Here p and ϕ are canonically-conjugated coordinates satisfying $\{p, \phi\} = -\mathrm{i}$, where $\{\cdot, \cdot\}$ are Poisson brackets.[1] For $g_1 = 0$ the system displays two classical fixed points: a stable one at $\phi = 0$ and an unstable one at $\phi = \pi$.

In the presence of a periodic drive ($g_1 \neq 0$), the unstable fixed point can become dynamically stable. This counterintuitive result was first obtained by Kapitza [23] in the high-frequency limit $\gamma^2 \gg g_0, g_1$.

[1] Throughout this paper we assume without loss of generality that $g_0 > 0$.