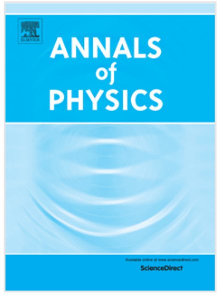

The precise time-dependent solution of the Fokker–Planck equation with anomalous diffusion

Guo Ran, Du Jiulin

PII: S0003-4916(15)00152-9
DOI: http://dx.doi.org/10.1016/j.aop.2015.04.019
Reference: YAPHY 66816

To appear in: *Annals of Physics*

Received date: 6 February 2015
Accepted date: 17 April 2015

The precise time-dependent solution of the Fokker-Planck equation with anomalous diffusion

Guo Ran, Du Jiulin*
Department of Physics, School of Science, Tianjin University, Tianjin 300072, China

Abstract We study the time behavior of the Fokker-Planck equation in Zwanzig's rule (the backward-Ito's rule) based on the Langevin equation of Brownian motion with an anomalous diffusion in a complex medium. The diffusion coefficient is a function in momentum space and follows a generalized fluctuation-dissipation relation. We obtain the precise time-dependent analytical solution of the Fokker-Planck equation and at long time the solution approaches to a stationary power-law distribution in nonextensive statistics. As a test, numerically we have demonstrated the accuracy and validity of the time-dependent solution.

Keywords: Time-dependent solution; Fokker-Planck equation; Anomalous diffusion; Brownian motion

1. Introduction

The Fokker-Planck (FP) equation was first applied to the Brownian motion problem [1]. With the equation of motion of a Brownian particle, Langevin equation, and the corresponding FP equation, the probability distribution to find the particle in a given region may be determined by solving the equation. The simplest situation of the Brownian motion is a Brownian particle moving in the medium with friction constant γ and diffusion constant D, and the link between the two constants is $D=\gamma kT$, known as the fluctuation-dissipation relation (FDR) [2]. In such a situation the Langevin equation and the FP equation are both linear and the solutions (stationary and time-dependent) are Gaussian distributions or Maxwell-Boltzmann (MB) distributions. But for a general situation when a Brownian particle is moving in a complex medium in which the friction and diffusion coefficient can depend on the variables, the Langevin equation is nonlinear and then solving the corresponding FP equation becomes very complicated. In fact, not much has been known in general about the long-time steady-state solution of an arbitrary FP equation. Only in some special cases if a FDR can be invoked, a steady-state solution is found.

The Brownian motion characterized as a pure diffusion process has a probability distribution that is Gaussian at all times and obeys the Einstein relation at long time, the mean-square displacement $<(\Delta x)^2>=2Dt$, where D is a constant, which is called normal diffusion. Anomalous diffusion is random motion having $<(\Delta x)^2> \sim t^{\nu}$ with $\nu\neq 1$ and therefore there is no constant diffusion coefficient (D may be space/velocity dependent [3-9]) and the associated probability distribution is non-Gaussian or non-MB/power-law distributions [10-17]. Many nonlinear FP equations which appear to be some "fractal structure" are frequently constructed to describe the systems

* Email address: jiulindu@aliyun.com