Annals of Physics 355 (2015) 48–54

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

On the Klein–Gordon oscillator subject to a Coulomb-type potential

K. Bakke *, C. Furtado

Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970, João Pessoa, PB, Brazil

HIGHLIGHTS

- Interaction between the Klein–Gordon oscillator and a modified mass term.
- Relativistic bound states for both attractive and repulsive Coulomb-type potentials.
- Dependence of the Klein–Gordon oscillator frequency on the quantum numbers.
- Relativistic analogue of a position-dependent mass system.

ARTICLE INFO

Article history:
Received 12 August 2014
Accepted 28 January 2015
Available online 7 February 2015

Keywords:
Klein–Gordon oscillator
Coulomb-type potential
Biconfluent Heun equation
Relativistic bound states

ABSTRACT

By introducing the scalar potential as modification in the mass term of the Klein–Gordon equation, the influence of a Coulomb-type potential on the Klein–Gordon oscillator is investigated. Relativistic bound states solutions are achieved to both attractive and repulsive Coulomb-type potentials and the arising of a quantum effect characterized by the dependence of angular frequency of the Klein–Gordon oscillator on the quantum numbers of the system is shown.

1. Introduction

In recent decades, the relativistic generalization of the harmonic oscillator has attracted a great deal of attention. The most known relativistic model of the harmonic oscillator was introduced by Moshinsky and Szczepaniak [1], which is known as the Dirac oscillator, and has been investigated

* Corresponding author.
E-mail addresses: kbakke@fisica.ufpb.br (K. Bakke), furtado@fisica.ufpb.br (C. Furtado).

http://dx.doi.org/10.1016/j.aop.2015.01.028

by several authors [2–11]. In particular, the Dirac oscillator has attracted a great interest in studies of Jaynes–Cummings model [8,4], the Ramsey-interferometry effect [9] and quantum phase transitions [10,11]. On the other hand, a relativistic model of the harmonic oscillator has also been proposed for a scalar particle by Bruce and Minning [12] based on the Dirac oscillator [1]. Bruce and Minning [1] showed that an analogous coupling to the linear Dirac oscillator coupling can be introduced into the Klein–Gordon equation in such a way that one can recover the Schrödinger equation for a harmonic oscillator in the nonrelativistic limit. This coupling proposed by Bruce and Minning is known as the Klein–Gordon oscillator [12–15]. For example, by considering the isotropic Klein–Gordon oscillator in $(2+1)$ dimensions, the Klein–Gordon equation becomes:

$$\left[\mathcal{E}^2 - m^2\right]\phi = \left[\hat{p} + im\omega\rho\,\hat{\rho}\right]\cdot\left[\hat{p} - im\omega\rho\,\hat{\rho}\right]\phi, \tag{1}$$

where m is the rest mass of the scalar particle, ω is the angular frequency of the Klein–Gordon oscillator, $\rho = \sqrt{x^2+y^2}$ and $\hat{\rho}$ is a unit vector in the radial direction. In recent years, the Klein–Gordon oscillator has been investigated in noncommutative space [16,17], in noncommutative phase space [18] and in $\mathcal{PT}$-symmetric Hamiltonian [19].

Recently, several authors have shown their interest in investigating relativistic effects [20–23] on systems where the motion of a particle is governed by harmonic oscillations, such as the vibrational spectrum of diatomic molecules [24], the binding of heavy quarks [25,26] and the oscillations of atoms in crystal lattices, by mapping them as a position-dependent mass system [27–30]. The importance of these potentials arises from the presence of a strong potential field. In particular, Bahar and Yasuk [20] dealt with the quark–antiquark interaction as a problem of a relativistic spin-0 particle possessing a position-dependent mass, where the mass term acquires a contribution given by a interaction potential that consists of a linear and a harmonic confining potential plus a Coulomb potential term. It is worth mentioning other works that have explored the relativistic quantum dynamics of a scalar particle subject to different confining potentials which can be in the interest of several areas of physics [31–36].

The aim of this work is to study the influence of a Coulomb-type potential on the Klein–Gordon oscillator. In recent years, the confinement of a relativistic scalar particle to a Coulomb potential has been discussed by several authors [37–41]. As discussed in Ref. [41], the procedure in introducing a scalar potential into the Klein–Gordon equation follows the same procedure in introducing the electromagnetic 4-vector potential. This occurs by modifying the momentum operator $p_\mu = i\partial_\mu$ in the form: $p_\mu \to p_\mu - qA_\mu(x)$. Another procedure was proposed in Ref. [42] by making a modification in the mass term in the form: $m \to m+S\left(\vec{r},\ t\right)$, where $S\left(\vec{r},\ t\right)$ is the scalar potential. This modification in the mass term has been explored in recent decades, for instance, by analysing the behaviour of a Dirac particle in the presence of static scalar potential and a Coulomb potential [43] and a relativistic scalar particle in the cosmic string spacetime [44]. In this work, we investigate the influence of a Coulomb-type potential on the Klein–Gordon oscillator by introducing the scalar potential as modification in the mass term in the Klein–Gordon equation. We obtain bound state solutions to the Klein–Gordon equation for both attractive and repulsive Coulomb-type potentials and show a quantum effect characterized by the dependence of angular frequency of the Klein–Gordon oscillator on the quantum numbers of the system, which means that not all values of the angular frequency are allowed.

The structure of this paper is as follows: in Section 2, we study the Klein–Gordon oscillator subject to a Coulomb-type potential in the Minkowski spacetime in $(2+1)$ dimensions; in Section 3, we present our conclusions.

2. Klein–Gordon oscillator under the influence of a Coulomb-type potential

In this section, we study the behaviour of the Klein–Gordon oscillator subject to a Coulomb-type potential in (2+1) dimensions. We consider the cylindrical symmetry, then, we write the line element of the Minkowski spacetime in the form (with $c = \hbar = 1$):

$$ds^2 = -dt^2 + d\rho^2 + \rho^2\,d\varphi^2. \tag{2}$$