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From Bessel beam to complex-source-point cylindrical wave-function

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HIGHLIGHTS

- Bessel beam is transformed into the non-paraxial cylindrical complex-source-point.
- Exact high-order tightly focused solutions are derived without any approximations.
- The exact solutions also satisfy the nonrelativistic Schrödinger equation.
- Electromagnetic beams are obtained as solutions of Maxwell's vectorial equations.
- Applications are in laser/electron beam imaging, tweezers, and radiation force.

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ABSTRACT

This investigation shows that a scalar Bessel beam can be transformed into the non-paraxial complex-source-point cylindrical wave (CSPCW). High-order CSPCW solutions, termed here high-order quasi-Gaussian cylindrical beams, which exactly satisfy the Helmholtz equation, are derived analytically. Moreover, partial-derivatives of the high-order CSPCW solutions satisfy the Helmholtz equation. In addition, the CSPCW solutions satisfy the nonrelativistic Schrödinger equation within standard quantum mechanics, thus, the results can be used in the description of elementary particle/matter motion and related applications in quantum scattering theory. Furthermore, the analysis is extended to the case of vector beams in which the components of the electromagnetic (EM) field are obtained based on different polarizations of the magnetic and electric vector potentials, which exactly satisfy Maxwell's vectorial equations and Lorenz' gauge condition. An attractive feature of the high-order solutions is the rigorous description of strongly focused (or strongly divergent) cylindrical wave-fields without any approximations, nor the need for numerical methods. Possible applications are in beam-forming design

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using high-aperture or collimated cylindrical laser/electron quasi-Gaussian beams in imaging microscopy, particle manipulation, optical tweezers, and the study of the scattering, and radiation forces on objects.

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1. Introduction

Bessel beams originate in the wave diffraction theory as an exact solution of the wave equation [1–4],

$$\Box^2 \Psi_i = 0, \tag{1}$$

where Ψ_i is a scalar wave function which describes the propagating field, and the d'Alembertian operator $\Box^2 = \nabla^2 - c^{-2}\partial/\partial t$, where c is the wave speed in the medium of wave propagation. For a Bessel beam (BB) propagating in a Cartesian coordinate system along the axial z direction, the generalized mathematical expression for the scalar field of vortex type is given by,

$$\begin{aligned} \Psi_i &= \Psi_{BB} \\ &= \Psi_0 J_m\left(k_\rho \rho\right) e^{i(k_z z + m\phi - \omega t)}, \end{aligned} \tag{2}$$

where Ψ_0 is the amplitude at the center of the coordinate system, $J_m(.)$ is the cylindrical Bessel function of order m, which determines the order of the beam. The fundamental solution ($m = 0$) has a bright spot at the center of the beam, whereas for high-orders, the center is a field null as expected from the high-order Bessel functions [4,5]. The parameter $\rho = \sqrt{x^2 + y^2}$ is the distance to a point in the transverse plane (x, y), $k_\rho = k \sin\beta$ and $k_z = k\cos\beta$ are the radial and axial wavenumbers, respectively, k is the wavenumber, β is the half-cone angle defined with respect to the axis of wave propagation z, such that $\beta = 0$ corresponds to plane waves propagating along z, the azimuthal angle is $\phi = \tan^{-1}(y/x)$ and the exponential $e^{-i\omega t}$ denotes the time-dependence where ω is the angular frequency.

Eq. (2) defines a BB of infinite extent, which is described by the product of two functions (excluding the time-dependence). One function, $\exp(ik_z z)$, describes the field's variations along the direction of wave propagation z, and the other function $J_m\left(k_\rho \rho\right)\exp(im\phi)$, describes the field's properties in the transverse plane, perpendicular to the propagation axis. Since Eq. (2) is separable, that is, when it can be written as a product of a transverse and an axial function, the beam has a long depth-of-field and the property to be nondiffracting (unaffected spreading), i.e., it propagates while its phase only (and not the amplitude) varies with z, but its transverse shape is unchanged [6,7]. In addition, a BB is self-healing [8–10]. That is, the beam reconstructs itself after encountering an obstruction as long as it propagates over multiple wavelengths and the whole beam is not totally blocked. Moreover, because of the axial null at the center of a high-order BB (i.e. $m \geq 1$), a particle or a multitude of particles can be trapped in the hollow region [11]. Similarly, the transverse shape or "slice" of a BB corresponds to a standing wave created by the superposition of two fields expressed by Hankel functions [12], in which a particle can be trapped. Also, a high-order BB with its phase varying according to $\exp(im\phi)$, carries an angular momentum [13–15] and exerts a force [16–19], and a torque [20–22] which sets an absorptive particle into rotation. It is the combination of these intriguing properties that has made BBs the focus of much research over the past decade and has poised them to make a great impact in the development of tweezers and particle manipulation.

In the following, it is demonstrated how a scalar (non-diffracting) Bessel beam can be transformed into the high-order complex-source-point [23–31] (CSP) cylindrical wave of order m. In contrast to Bessel beams, the high-order cylindrical quasi-Gaussian solution possesses a beam waist and displays tightly focused or (quasi-)collimated profiles depending on the beam's parameters. Partial-derivatives (PDs) as well as possible superposition of the fundamental or the PDs solutions are also discussed. It is shown that the proposed solutions satisfy the nonrelativistic Schrödinger equation within standard

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