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Separation of acoustic waves in isentropic flow perturbations

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HIGHLIGHTS

- First splitting of non-uniform flows in acoustic and non-acoustic components.
- These result leads to a generalisation of sound which is compatible with Lighthill's acoustic analogy.
- A closed equation for the generation and propagation of sound is given.

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ABSTRACT

The present contribution investigates the mechanisms of sound generation and propagation in the case of highly-unsteady flows. Based on the linearisation of the isentropic Navier–Stokes equation around a new pathline-averaged base flow, it is demonstrated for the first time that flow perturbations of a non-uniform flow can be split into acoustic and vorticity modes, with the acoustic modes being independent of the vorticity modes. Therefore, we can propose this acoustic perturbation as a general definition of sound.

As a consequence of the splitting result, we conclude that the present acoustic perturbation is propagated by the convective wave equation and fulfils Lighthill's acoustic analogy. Moreover, we can define the deviations of the Navier–Stokes equation from the convective wave equation as "true" sound sources.

In contrast to other authors, no assumptions on a slowly varying or irrotational flow are necessary.

Using a symmetry argument for the conservation laws, an energy conservation result and a generalisation of the sound intensity are provided.

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1. Introduction

Nowadays, several manufacturers have to pay increasingly attention to the noise of highly-unsteady flows. On the one hand, there are many applications where the background noise disturbs the desired acoustic signal. On the other hand, the acoustic comfort has become an important selection criteria for consumers. Therefore, it is surprising that the following significant question is open until now:

> Is it possible to derive a closed system of equations for the acoustic quantities in the case of highly-unsteady flows?

Hence, the general mechanisms of sound generation and propagation are also unknown. The qualitative definition of an acoustic perturbation is widely accepted being that part of fluctuation which is radiating with a velocity that depends on the speed of sound, whereas non-acoustic perturbations are convected by the hydrodynamic flow. To the best of the author's knowledge, there exists no decomposition of the highly-unsteady flow variables, where the acoustic quantity with respect to this definition satisfies a closed system of equations. The following remark captures this situation [1, p. 283]:

> It is impossible to identify the "sources" of sound without first defining what is actually meant by "sound". Unfortunately, current understanding of unsteady compressible flows, especially turbulent shear flows, is still too rudimentary to give a completely general definition of this quantity.

The aim of the paper is to give an answer to the open question raised in the opening paragraph. More precisely we present a mathematical analysis where the underlying decomposition of the flow variables is necessary for the key arguments. By virtue of the fact that the acoustic fluctuations satisfy a closed equation, this pressure fluctuation is convenient for a general definition of sound.

Moreover, the acoustic pressure fluctuation may be interpreted as the solution of Lighthill's analogy and of a convective wave equation with a true sound source.

As usual, the following steps will be considered:

1. splitting the variables of the compressible Navier–Stokes equations ρ, u, p into their base flow $\bar{(\cdot)}$ and their fluctuations $(\cdot')$;
2. linearising around the base flow;
3. neglecting the dissipation mechanisms for the fluctuations;
4. separating the acoustic fluctuations from the non-acoustic perturbations.

Motivated by the large disparity between the energy of the hydrodynamic and acoustic variables, an assumption like

$$\frac{|\rho'|}{|\bar{\rho}|} = O(\epsilon), \qquad \frac{\|u'\|}{\|\bar{u}\|} = O(\epsilon), \qquad \frac{|p'|}{|\bar{p}|} = O(\epsilon), \quad 0 < \epsilon \ll 1, \tag{1}$$

ensures a small linearisation error. Because of the conservation form of the Navier–Stokes equations one may expect a resulting linear conservation law after step three. However, conservation laws for the fluctuation components are only known in special cases, e.g. in the case of uniform mean flows w.r.t. time averaging. Moreover, an exact separation of acoustic waves from other compressibility effects is also known exclusively in this special case (cf. [2, Splitting Theorem, p. 220]). Therefore, the key motivation of the paper was the following question:

> Can we define a base flow such that the equations for the perturbations can be formulated as a conservation law and what are the consequences for the separation of the acoustic waves?

It turns out that it is convenient to work with a base flow which is constant along the pathlines of the fluid:

$$\partial_t \bar{p} + u \cdot \nabla \bar{p} = 0, \qquad \partial_t \bar{u} + u \cdot \nabla \bar{u} = 0.$$

Namely, sound waves are interpreted as vibrations around the fluid particle path. Because of $\nabla \bar{u} = 0$ in the case of constant base flow, the associated momentum equation implies $\nabla \bar{p} = 0$. Obviously,