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Concise analytic solutions to the quantum Rabi model with two arbitrary qubits

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ABSTRACT

Using extended coherent states, an analytical exact study has been carried out for the quantum Rabi model (QRM) with two arbitrary qubits in a very concise way. The *G*-functions with 2×2 determinants are generally derived. For the same coupling constants, the simplest *G*-function, resembling that in the one-qubit QRM, can be obtained. Zeros of the *G*-function yield the whole regular spectrum. The exceptional eigenvalues, which do not belong to the zeros of the *G* function, are obtained in the closed form. The Dark states in the case of the same coupling can be detected clearly in a continued-fraction technique. The present concise solution is conceptually clear and practically feasible to the general two-qubit QRM and therefore has many applications.

1. Introduction

Quantum Rabi model (QRM) describes a two-level atom (qubit) coupled to a cavity electromagnetic mode (an oscillator) [1], a minimalist paradigm of matter–light interactions with applications in numerous fields ranging from quantum optics, quantum information science to condensed matter physics. The solutions to the QRM are however highly nontrivial. Whether an analytical exact solution even exists is uncertain for a long time. Recently, Braak presented an analytical exact solution [2] to the QRM using the representation of bosonic creation and annihilation operators in the Bargmann space of analytical functions [3]. A so-called *G*-function with a single energy variable was derived yielding

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exact eigensolutions, which is not in the closed form but well defined mathematically. Alternatively, using the method of extended coherent states (ECS), this G-function was recovered in a simpler, yet physically more transparent manner by Chen et al. [4]. Braak's solution has stimulated extensive research interests in the single-qubit QRM [5].

As quantum information resources, such as the quantum entanglement [6] and the quantum discord [7], can be easily stored in two qubits, two qubits in a common cavity have potential applications in quantum information technology. Such a model system with two qubits now can be constructed in several solid devices [8–10]. Recently, some analytical studies to the QRM with two qubits have been attempted within various approaches [11–17]. By using the ECS technique, a G-function for the QRM with two equivalent qubits, resembling the simplest one without a determinant in the single-qubit QRM [2], was obtained [15]. While in the Bargmann representation, the G-function was built as a high order determinant, such as 8×8 determinants for QRM with two different qubits [12–14].

Practically, the QRM with two arbitrary qubits is the most general one, and can be realized in experimental device with the greatest possibility. We believe that a simpler G-function is more convenient to obtain the eigensolutions, and can also shed light on many physical processes more clearly. In this work, employing the ECS, we demonstrate a successful derivation of a very concise G-function, which is just a 2×2 determinant for the QRM with two arbitrary qubits. Furthermore, for the same coupling, the G-function even for two different qubits can be reduced to a simplest one without the use of the determinant, like that in the single-qubit QRM [2].

2. Analytical scheme to exact solutions

The Hamiltonian of the QRM with two qubits can be generally written as [12–14]

$$H = \omega d^\dagger d + g_1\sigma_{1x}(d^\dagger + d) + g_2\sigma_{2x}\,(d^\dagger + d) + \Delta_1\sigma_{1z} + \Delta_2\sigma_{2z}, \tag{1}$$

where $\Delta_i (i = 1, 2)$ is the energy splitting of the ith qubit, $d^\dagger$ creates one photon in the common single-mode cavity with frequency ω, g_i describes the coupling strength between the ith qubit and the cavity, σ_{ix} and σ_{iz} are the usual Pauli matrices of the ith qubit. After a rotation with respect to the y-axis by an angle $\frac{\pi}{2}$, the Hamiltonian in the two-qubit basis $|1, 1\rangle$, $|1, -1\rangle$, $|-1, 1\rangle$, and $|-1, -1\rangle$, which are eigenstates of $\sigma_{1z}\otimes\sigma_{2z}$, can be written as the following symmetric matrix (in unit of $\omega = 1$)

$$H = \begin{pmatrix} d^\dagger d + g\left(d^\dagger + d\right) & -\Delta_2 & -\Delta_1 & 0 \\ -\Delta_2 & d^\dagger d + g'\left(d^\dagger + d\right) & 0 & -\Delta_1 \\ -\Delta_1 & 0 & d^\dagger d - g'\left(d^\dagger + d\right) & -\Delta_2 \\ 0 & -\Delta_1 & -\Delta_2 & d^\dagger d - g\left(d^\dagger + d\right) \end{pmatrix}, \tag{2}$$

where $g = g_1 + g_2$ and $g' = g_1 - g_2$.

For later use, we express the wavefunction in terms of the Fock space as

$$|d\rangle = \sum_{n=0}^{\infty}\sqrt{n!}\left\{a_n\left[|1, 1\rangle \pm (-1)^n\,|-1, -1\rangle\right] + b_n\left[|1, -1\rangle \pm (-1)^n\,|-1, 1\rangle\right]\right\}|n\rangle, \tag{3}$$

where $+(-)$ corresponds to even (odd) parity, $|n\rangle$ is the photonic number state. The Schrödinger equation leads to the recurrence relation

$$a_{m+1} = \frac{\left[\Delta_2 \pm \Delta_1 (-1)^m\right] b_m - (m-E)\,a_m - g a_{m-1}}{g\,(m+1)},$$
$$b_{m+1} = \frac{\left[\Delta_2 \pm \Delta_1 (-1)^m\right] a_m - (m-E)\,b_m - g' b_{m-1}}{g'\,(m+1)}. \tag{4}$$

Note that they cannot be reduced to a linear three-term recurrence form. The coefficients a_n, b_n can be obtained in terms of two initial values of a_0 and b_0 recursively.

In this paper, we will first study the general case of different coupling strengths with the same cavity, then we turn to the special equal coupling case.