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Conformal geometrodynamics regained: Gravity from duality

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ABSTRACT

There exist several ways of constructing general relativity from 'first principles': Einstein's original derivation, Lovelock's results concerning the exceptional nature of the Einstein tensor from a mathematical perspective, and Hojman–Kuchař-Teitelboim's derivation of the Hamiltonian form of the theory from the symmetries of space–time, to name a few. Here I propose a different set of first principles to obtain general relativity in the canonical Hamiltonian framework without presupposing space–time in any way. I first require consistent propagation of scalar spatially covariant constraints (in the Dirac picture of constrained systems). I find that up to a certain order in derivatives (four spatial and two temporal), there are large families of such consistently propagated constraints. Then I look for pairs of such constraints that can gauge-fix each other and form a theory with two dynamical degrees of freedom per *space* point. This demand singles out the ADM Hamiltonian either in (i) CMC gauge, with arbitrary (finite, non-zero) speed of light, and an extra term linear in York time, or (ii) a gauge where the Hubble parameter is conformally harmonic.

1. Introduction

In the golden years of the canonical approach to general relativity, one of the most profound thinkers on gravity, John Wheeler, posed the famous question [1]: "If one did not know the Einstein–Hamilton–Jacobi equation, how might one hope to derive it straight off from plausible first principles, without ever going through the formulation of the Einstein field equations themselves?".

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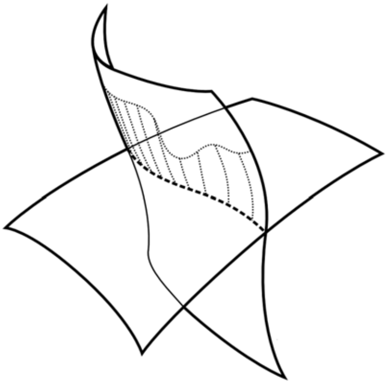

Fig. 1. First-class constraint surfaces in phase space intersecting transversally. The gauge orbits of one of the constraints are indicated by the dotted lines emanating from the intersection, where the reduced theory lies.

In response, Hojman, Kuchař and Teitelboim (HKT), in the aptly titled paper "Geometrodynamics regained" [2], tackled the problem of deriving geometrodynamics directly from first principles rather than by a formal rearrangement of Einstein's law. They succeeded in obtaining the canonical form of general relativity by imposing requirements onto a Hamiltonian formulation ensuring that it represents a foliated space–time.

Here I propose a different approach to Wheeler's question. In short, I will look for what I refer to as *dual symmetries* in the gravitational phase space. Dual symmetries consist of two distinct sets of constraints, which I refer to as *the dual partners*. Each dual partner should be first of all a (spatially covariant) first class constraint – which by Dirac's analysis means that each generates a (spatially covariant) symmetry – and secondly, to fix the partnership and establish duality, one partner must gauge-fix the other. In other words, dual symmetries should be ones for which one can always find a compatible space of observables. In Fig. 1, we see two first class constraint manifolds intersecting transversally, which illustrates the rather simple principle. Alternatively, this criterion amounts to searching for spatially covariant theories with two propagating degrees of freedom, which possess a gauge-fixing that is also consistently propagated. This dual role arises because in the Hamiltonian formalism, a gauge-fixing condition – represented as the vanishing of a given phase space function – also generates a transformation in phase space (the symplectic flow of said function).

The deeper reason for taking these first principles as the basis of my construction cannot, however, be fully appreciated by only considering the classical theory. As realized in the mid 60's by Feynman, and later resolved simultaneously by Becchi, Rouet, Stora and Tyutin [3,4], theories with non-abelian symmetries require extra care upon quantization, so that pure gauge degrees of freedom do not propagate. The extended theoretical framework in which these redundancies are adequately taken into account is today known as BRST. In the Hamiltonian setting, the conditions required here for dual constraints imbue the extended BRST system with interesting properties. Namely, they ensure that with just the right gauge-fixing, the gauge-fixed, BRST-extended Hamiltonian possesses not only the symmetries related to the original system, but also those related to its gauge-fixing. Thus the results obtained here can be said to emerge out of broad symmetry requirements, but it is surprising that we do not have to demand in advance what symmetries the emerging theory should embody, they are, in a weak sense, self-selected.

In their seminal paper, Hojman–Kuchař–Teitelboim [2] used the fact that the set of vector fields that generate the tangential and normal deformations of an embedded hypersurface in a Riemannian manifold produce a specific vector commutation algebra, i.e. a specific symmetry. They then sought constraints in the space of functionals of the spatial metric g_{ab} and momenta π^{ab}, whose own commutation algebra (Poisson bracket) would mirror the hypersurface deformation algebra. Clearly, this derivation must assume the prior existence of space–time. With a few other requirements, they were eventually led to the (super)momentum constraint $H_a(x) = \nabla_b\pi^b{}_a(x) = 0$ and the (super)