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Role of quantum statistics in multi-particle decay dynamics

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ABSTRACT

The role of quantum statistics in the decay dynamics of a multi-particle state, which is suddenly released from a confining potential, is investigated. For an initially confined double particle state, the exact dynamics is presented for both bosons and fermions. The time-evolution of the probability to measure two-particle is evaluated and some counterintuitive features are discussed. For instance, it is shown that although there is a higher chance of finding the two bosons (as oppose to fermions, and even distinguishable particles) at the initial trap region, there is a higher chance (higher than fermions) of finding them on two opposite sides of the trap *as if the repulsion between bosons is higher than the repulsion between fermions*. The results are demonstrated by numerical simulations and are calculated *analytically* in the short-time approximation. Furthermore, experimental validation is suggested.

1. Introduction

The scenario, in which particles are released from confining potentials, is common in recent theoretical and experimental studies [1–7]. In these experiments, particles are initially localized in a spatial trap and then they are released and propagate freely. This well-known experimental setup can emulate other physical scenarios, such as the tunneling decay of alpha particles from a radioactive nucleus, the emission of a particle from a molecule and, of course, the slit experiment. Moreover, recent

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advances in atom cooling and trapping technologies make it possible to trap bosonic and fermionic gases [8–17] or even low atom-number (Fock) states [18] by optical means. These optical traps can be shut off almost instantaneous and therefore allow investigating the subsequent expansion dynamics.

Consequently, the issue of particles releasing received a lot of theoretical attention in the literature. The subject was investigated for multiple (N) particles [19], in a double well [20], in the presence of a nonlinear potential [21], in tunneling scenarios [22], and even with Coulomb interaction [23]. However, to the best of our knowledge, the dependence of a correlative escape on the symmetry of the wavefunction was not investigated.

The escape of several indistinguishable particles from a trap is particularly interesting since it was usually assumed that indistinguishable particles have a tendency to bunch (in case of bosons) or anti-bunch (in the case of fermions). This tendencies of bosons bunching and fermions anti-bunching were well documented in the literature [24–27]. Therefore, naively one would expect that fermions would prefer to leave the trap, while bosons would prefer to stick together and to remain within the trap, or, at least, to leave the trap in the same direction.

Recently, however, it was shown [28] that in some cases, when one of the wavefunctions has a zero then the opposite can occur: bosons (will) anti-bunch while fermions tend to bunch.

In particular it was demonstrated that in such a scenario the probability of two adjacent detectors (on two sides of the zero) to measure simultaneously two fermions is twice as large as the probability to measure two distinguishable particles there. Moreover, the probability to measure two bosons there is *zero*. These results are counter-intuitive and in fact contradict the bosons bunching and fermions anti-bunching premises.

In every symmetric trap this scenario is very common since half of the eigenstates are anti-symmetric and half are symmetric. Therefore, the obvious question arises: if a given initial condition (e.g., one state symmetric and the other anti-symmetric) determines local fermion (or bosons) bunching (or anti bunching), what would be the effect on their escape probability.

It is the object of this paper to investigate the *generic* escaping dynamics of pairs of particles. We investigate the two cases when the escaping particles are either bosons or fermions and compare between them.

It should be stressed that while most of the paper's conclusions are totally generic, and can be apply to any symmetric trap, we have decided to focus in this paper on infinite well, where the contrast between fermions and bosons are enhanced dramatically.

2. Generic trap

To simulate the trapping process we define a spatial regime $|x| < a$, which can be regarded as a trap. Initially we assume that both particles are located in the trap, i.e., at $|x| < a$; hence, we can assume that the Schrödinger wave equation

$$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2} + V(x,t)\,\psi \tag{1}$$

(where the dimensionless units $\hbar = 1$ and $2m = 1$ were adopted) is initially governed by the potential

$$V(x, t < 0) = \begin{cases} V_{in}(x) & |x| < a \\ \infty & |x| \geq a \end{cases} \tag{2}$$

where $V_{in}(x)$ is an arbitrary potential.

However, for $t > 0$ the particles are released by reducing the trap's boundaries to

$$V(x, t \geq 0) = V_{in}(x) \tag{3}$$

and the particles are free to escape from the trap to infinity.

The particular case where initially $V_{out} = \infty$, $V_{in} = 0$, and for $t > 0$ $V_{out} = V_{in} = 0$ is discussed in details below, but it should be stressed that the main conclusions are totally generic.

Now suppose that $\psi_1(x, t = 0)$ and $\psi_2(x, t = 0)$ are both eigenstates of the initial Schrödinger equation (i.e., with the initial potential equations (1) and (2)), and $\psi_1(x, t)$ and $\psi_2(x, t)$ are their