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An analytical shear-lag model for composites with 'brick-and-mortar' architecture considering non-linear matrix response and failure

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ABSTRACT

Discontinuous composites can combine high stiffness and strength with ductility and damage tolerance. This paper presents an analytical shear-lag model for the tensile response of discontinuous composites with a 'brick-and-mortar' architecture, composed of regularly staggered stiff platelets embedded in a soft matrix. The formulation is applicable to different types of matrix material (e.g. brittle, perfectly-plastic, strain-hardening), which are modelled through generic piecewise-linear and fracture-mechanics consistent shear constitutive laws. Full composite stress–strain curves are calculated in less than 1 second, thanks to an efficient implementation scheme based on the determination of process zone lengths. Parametric studies show that the model bridges the yield-slip (plasticity) theory and fracture mechanics, depending on platelet thickness, platelet aspect-ratio and matrix constitutive law. The potential for using 'brick-and-mortar' architectures to produce composites which are simultaneously strong, stiff and ductile is discussed, and optimised configurations are proposed.

1. Introduction

Most natural structural materials combining high stiffness, high strength and damage tolerance (e.g. nacre, bone and spider silk) share a common motif: a discontinuous *'brick-and-mortar'* architecture (see Fig. 1a) with staggered stiff inclusions (e.g. fibres or platelets) embedded in a soft matrix [1,2]. This provides two deformation mechanisms under tension: (i) extension of the inclusions (which dominates in the elastic domain and confers initial stiffness), and (ii) shearing of the matrix (which promotes large deformations and energy dissipation before failure). It is suggested that the combination of these two mechanisms in optimised architectures is key to achieving the impressive performance of many natural composites.

In contrast to natural composites, high-performance Fibre Reinforced Polymers (FRPs) typically use continuous fibres, thus achieving high stiffness and strength but presenting limited ductility. Mimicking the discontinuous architecture of natural composites could potentially overcome this limitation and extend the applicability of FRPs to damage tolerant structures. This requires designing the material microstructure, and thus modelling the effect of discontinuities on the response of composites [3,4].

One of the most widespread models for discontinuous composites is the Kelly–Tyson yield-slip theory [5]. This assumes that the matrix is perfectly-plastic and transfers stresses between the inclusions by yielding under shear; the performance of the composite is therefore governed by the matrix's shear strength S^{m}. For relatively low aspect-ratio inclusions and neglecting the thickness of the matrix, the strength of the composite X_S^{∞} is related to the overlapping inclusion length l^{b} and inclusion thickness t^{b} (see Fig. 1) by:

$$X_S^{\infty} = l^{b} \cdot S^{m} / t^{b}. \tag{1}$$

This assumes that the inclusions withstand the tensile stresses required to yield the matrix in shear (i.e. the tensile strength of the inclusions is $X^{b} \geqslant 2 \cdot X_S^{\infty}$). The optimal inclusion geometry is therefore defined by a *critical overlapping length* $l_{crit}^{b} = X^{b} \cdot t^{b} / (2 \cdot S^{m})$.

An alternative to the *plasticity* or *strength*-based approach in Eq. (1) is a *fracture mechanics* or *toughness*-based formulation, which has been applied to discontinuous FRPs with brittle matrices [6,7]. Assuming that the composite fails when a mode-II crack propagates in the matrix from the ends of the inclusions inwards, and neglecting the effect of friction, the strength of the composite depends on the matrix's (or matrix–inclusion interface's) mode-II fracture toughness $\mathcal{G}_{IIc}^{m}$ through:

$$X_{\mathcal{G}}^{\infty} = \sqrt{2 \cdot E^{b} \cdot \mathcal{G}_{IIc}^{m} / t^{b}}. \tag{2}$$

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Nomenclature

Uppercase roman variables

$\mathcal{A}$	platelet $\mathcal{A}$
$\mathcal{B}$	platelet $\mathcal{B}$
E	tensile stiffness
G	shear stiffness
$\mathcal{G}_c$	critical energy release rate (fracture toughness)
L	characteristic length
N	total number of matrix subdomains
S	shear strength
T	characteristic thickness
V	volume fraction
X	tensile strength

Lowercase roman variables

l	length
e	tensile failure strain
ℓ	length of matrix subdomain/process zone
n	number of non-central active subdomains
t	thickness
$\mathbf{s}$	subdomains vector
u	displacement
x	location along overlap

Lowercase greek variables

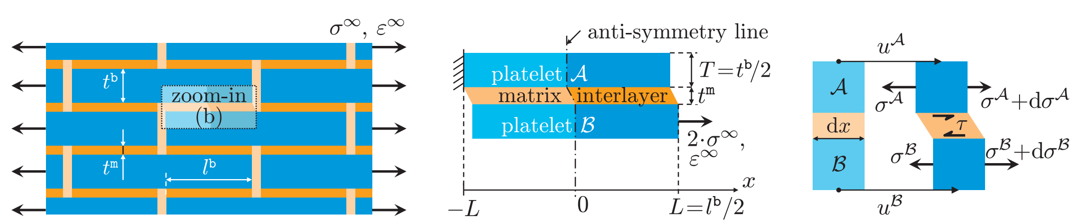
α	characteristic aspect ratio, $\alpha = L/T$
ε	tensile strain
γ	shear strain
λ	characteristic coefficient, Eq. (5)
σ	longitudinal stress
$\Delta\sigma$	difference in platelet stresses, $\Delta\sigma = \sigma^{\mathcal{B}} - \sigma^{\mathcal{A}}$
τ	shear stress

Superscripts

$\mathtt{b}$	platelet/inclusion ('brick')
$[i]$	matrix subdomain
$\mathtt{m}$	matrix ('mortar')
$\mathtt{pz}$	process zone (matrix damage)
∞	remote
$\bigstar$	ideal geometry for a brittle matrix

Subscripts

II	mode-II delamination
$\mathtt{M}$	macroscopic response
$\mathtt{uc}$	unit-cell response
$\mathtt{un}$	unloading response



(a) Composite with 'brick–and–mortar' architecture. (b) Unit–cell (zoom-in from (a)). (c) Infinitesimal element.

Fig. 1. Model overview.

Eqs. (1) and (2) represent two apparently contradictory criteria whose applicability has been largely debated in the literature [8–13]. It is generally accepted that the former is suitable for ductile matrices (with strain at the ultimate stress above 50%) and the latter for brittle ones (with strain at the ultimate stress below 10%), although the exact ductile-to-brittle transition is yet to be defined. Moreover, Bazant's theory for size effects in quasi-brittle materials [14] suggests that the size of the inhomogeneities relatively to that of the damage process zone also plays a role on the applicability of strength- and toughness-based criteria.

In addition, some details of the matrix's response (e.g. constitutive or geometric strain-hardening) are considered to be fundamental for the outstanding response of some natural composites [3,4,15], but are not accounted for in either strength- or toughness-based formulations. Altogether, a more comprehensive modelling framework is required to understand the influence of varying the matrix constitutive law and the geometry of the inclusions, as well as to predict the entire stress–strain curve of discontinuous composites.

The structured architecture of perfectly staggered discontinuous composites allows for the definition of reduced unit-cells, which simplifies their analysis significantly. However, and despite extensive work in modelling composites with 'brick-and-mortar' architecture [3,4,15,16], no formulation in the literature is able to cope with a generic range of inclusion sizes and a generic matrix constitutive law including failure.

This paper presents a model for the influence of discontinuities on the response of composites, depending on the dimensions of the inclusions—hereafter referred to as *platelets*—and matrix shear response. Section 2 develops a new shear-lag analytical model for perfectly staggered discontinuous composites, considering a piecewise linear but otherwise generic matrix constitutive law (including non-linearity and fracture). Section 3 validates analytical results through Finite Elements (FE) analyses, examines local stress fields and the global composite's response, and presents parametric studies on platelet geometry and the matrix's constitutive law. Section 4 discusses the model and its results, its relation with existing literature, and how it can be used to develop improved composites. Finally, Section 5 summarises the main conclusions.

2. Model development

2.1. Shear-lag formulation

Consider the 2D composite with 'brick-and-mortar' architecture represented in Fig. 1a), composed of stiff *platelets* (identified by the