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Dirac oscillator in perpendicular magnetic and transverse electric fields

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HIGHLIGHTS

- We study Dirac Oscillator with magnetic as well as electric field.
- Exact solutions are found.
- Critical cases have been examined.

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ABSTRACT

We study $(2 + 1)$ dimensional massless Dirac oscillator in the presence of perpendicular magnetic and transverse electric fields. Exact solutions are obtained and it is shown that there exists a critical magnetic field B_c such that the spectrum is different in the two regions $B > B_c$ and $B < B_c$. The situation is also analyzed for the case $B = B_c$.

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1. Introduction

The Dirac oscillator is one of the few interactions for which the Dirac equation is exactly solvable [1]. Apart from the intrinsic interest the Dirac oscillator and various related models have been studied extensively [2] because of their probable applications in many branches of physics, e.g., in optics [3], graphene [4], Jaynes–Cummings model [5] etc. Recently it has also been realized experimentally in microwaves [6]. An interesting model emerges when the Dirac oscillator is subjected to a

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homogeneous perpendicular magnetic field. This system too is exactly solvable [7]. A particularly important feature of this system is that it exhibits quantum chirality phase transition [8]. That is, there exists a critical magnetic field B_c such that the system exhibits a chirality quantum phase transition when $B > B_c$ and the spectrum is different in the two regions $B > B_c$ and $B < B_c$ [8]. In this context we would like to point out that the origin of relativistic Landau problem and the Dirac oscillator is entirely different—in the former case the magnetic field is introduced via minimal coupling while in the latter case the interaction is introduced via non minimal coupling and can be viewed as anomalous magnetic interaction.

It may be noted that the massless $(2 + 1)$ dimensional Dirac equation is exactly solvable in the presence of a homogeneous perpendicular magnetic and a transverse homogeneous electric field [9–11]. Such a system plays an important role e.g, in the context of Hall effect in graphene [11]. Here we shall examine the same system as in Ref. [11] but in addition we would also incorporate an oscillator type interaction. It will be shown that this model is exactly solvable and there exists a critical magnetic field B_c such that the spectra in the regions $B > B_c$ and $B < B_c$ are different. Furthermore the critical point $B = B_c$ is actually a point of discontinuity of the spectrum. The present problem can also be viewed in a different way. It is known that the Dirac oscillator problem remains exactly solvable in the presence of a homogeneous magnetic field [7]. It will be seen that it remains so even when a homogeneous transverse electric field is introduced. In particular we shall analyze in detail the spectrum for $B > B_c, B < B_c$ as well as for $B = B_c$. It will also be shown that when the magnetic field is turned off the problem (Dirac oscillator in the presence of an electric field) still admits exact bound state solutions provided the oscillator strength (k) is greater than a certain critical value (k_c). The organization of the paper is as follows : in Section 2 we formulate the problem; in Sections 3–5 we obtain solutions in different regimes of the magnetic field; in Section 6 we analyze the case when $c|\nu| = |E|$ where ν and E denote the effective magnetic and electric field strength respectively; finally Section 7 is devoted to a conclusion.

2. Formulation of the problem

The $(2+1)$ dimensional massless Dirac equation in the presence of a perpendicular magnetic and a transverse electric field is given by [11]

$$c\left[\sigma_x P_x + \sigma_y P_y\right]\Psi - \mathbf{e}V\Psi = \epsilon\Psi \tag{1}$$

where c denotes the velocity of light, $\mathbf{e}$ denotes the electric charge and $\Psi = \binom{\Psi_1}{\Psi_2}$ is a two component spinor. The generalized momenta and the electric field are given by

$$P_x = p_x + \mathbf{e}A_x, \qquad P_y = p_y + \mathbf{e}A_y, \qquad A_x = -By, \qquad A_y = 0, \qquad V = Ey. \tag{2}$$

It will be seen that it is possible to introduce an oscillator interaction in Eq. (1), still keeping the resulting equation solvable. Eq. (1) in the presence of an oscillator interaction in the same direction as the electric field is given by

$$c(\sigma_x P_x + \sigma_y P_y - iky\sigma_y\sigma_z)\Psi - \mathbf{e}Ey\Psi = \epsilon\Psi. \tag{3}$$

We shall now obtain exact solutions of Eq. (3) and examine how the spectrum depends on the magnetic and electric fields as well as the oscillator interaction. More precisely it will be seen that there exists a critical value of the magnetic field, namely, $B_c = \frac{k}{\mathbf{e}}$ such that the solutions are different for $B > B_c$ and $B < B_c$.

3. Weak magnetic field : $B < B_c$

In this case Eq. (3) can be written as

$$c\begin{pmatrix} -\dfrac{\mathbf{e}}{c}Ey & p_x - ip_y + \mathbf{e}\nu y \\ p_x + ip_y + \mathbf{e}\nu y & -\dfrac{\mathbf{e}}{c}Ey \end{pmatrix}\Psi = \epsilon\Psi \tag{4}$$

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