Annals of Physics 351 (2014) 112–137

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Local quanta, unitary inequivalence, and vacuum entanglement

Matías R. Vázquez, Marco del Rey, Hans Westman *, Juan León

Instituto de Física Fundamental, CSIC, Serrano 113-B, 28006, Madrid, Spain

ARTICLE INFO

Article history:
Received 21 March 2014
Accepted 28 July 2014
Available online 7 August 2014

Keywords:
Vacuum entanglement
Reeh–Schlieder theorem
Unitary inequivalence
Local particles

ABSTRACT

In this work we develop a formalism for describing localised quanta for a real-valued Klein–Gordon field in a one-dimensional box $[0, R]$. We quantise the field using *non-stationary local modes* which, at some arbitrarily chosen initial time, are completely localised within the left or the right side of the box. In this concrete set-up we directly face the problems inherent to a notion of local field excitations, usually thought of as elementary particles. Specifically, by computing the Bogoliubov coefficients relating local and standard (global) quantisations, we show that the local quantisation yields a Fock representation of the Canonical Commutation Relations (CCR) which is *unitarily inequivalent* to the standard one. In spite of this, we find that the local creators and annihilators remain well defined in the global Fock space $\mathfrak{F}^G$, and so do the local number operators associated to the left and right partitions of the box. We end up with a useful mathematical toolbox to analyse and characterise local features of quantum states in $\mathfrak{F}^G$. Specifically, an analysis of the global vacuum state $|0_G\rangle \in \mathfrak{F}^G$ in terms of local number operators shows, as expected, the existence of entanglement between the left and right regions of the box. The local vacuum $|0_L\rangle \in \mathfrak{F}^L$, on the contrary, has a very different character. It is neither cyclic (with respect to any local algebra of operators) nor separating and displays no entanglement between left and right partitions. Further analysis shows that the global vacuum also exhibits a distribution of local excitations reminiscent, in some respects, of a thermal bath. We discuss how

* Corresponding author. Tel.: +34 630847079.
E-mail addresses: mrodvaz@gmail.com (M.R. Vázquez), marco.del.rey@iff.csic.es (M. del Rey), hwestman74@gmail.com (H. Westman), juan.leon@csic.es (J. León).

http://dx.doi.org/10.1016/j.aop.2014.07.031

the mathematical tools developed herein may open new ways for the analysis of fundamental problems in local quantum field theory.

1. Introduction

Quantum Field Theory (QFT in short) has proven to be one of the most successful theories in Physics. Its potential to describe the properties of elementary particles has been richly demonstrated within the framework of the Standard Model of Particle Physics. The extraordinary agreement between theoretical and experimental values of the muon $g-2$ anomaly [1], or the recent experimental success vindicating the Higgs mechanism after decades of search [2,3], are just two examples among many.

Elementary particles in modern physics are commonly thought of as small localised entities moving around in space. A careful examination, however, reveals such an interpretation to be problematic: in QFT a free particle is represented by a superposition of positive-frequency complex-valued modes which satisfy some field equation (e.g. the Klein–Gordon equation). Yet, no superposition of positive-frequency modes can be localised within a region of space, even for an arbitrarily small period of time [4].

This confusing issue is sometimes mistaken as superluminality, see [5] for a clarification. In fact, it can be shown that the time derivative $\dot{\psi}$, for any wave-packet ψ composed exclusively out of positive frequency modes, is non-zero almost everywhere in space.[1] For that reason, even if ψ propagates in a perfectly causal manner according to the Klein–Gordon equation, it can hardly represent a localised entity. It is problematic to think of the fundamental field excitations of QFT as 'particles' in any common sense of the word.

The problem of localisation can be analysed from other angles, for example in terms of *localisation systems*. These are defined in terms of a set of projectors E_Δ on bounded spatial regions Δ whose expectation values yield the probability of a position measurement to find the particle within Δ. A theorem by Malament [7] shows that in a Minkowski spacetime, under reasonable assumptions for the projector algebra, no such non-trivial set of projectors exists. There is also a general result (valid for both, relativistic or non-relativistic cases) due to Hegerfeldt [4] proving that, assuming a Hamiltonian with spectrum bounded from below, the expectation value of those projectors is non-zero for almost all times. In particular this applies also to states naively thought to be localised. Also along this line, but in order to describe unsharp localisation systems, Busch [8] replaced the use of projectors by more general operators, "effects" (or Positive-Operator Valued Measures—POVM), showing that it is impossible to localise with certainty a particle in any bounded region of space. Furthermore, completing the collection of no-go theorems, Clifton and Halvorson [9] have shown, under a set of natural requirements, that it is not possible to define local number operators associated to any finite region of space. At this point it is also worthwhile mention the well-known problems of other efforts, based on the use of putative observables such as the Newton–Wigner position operator [10–12].

In addition, there is also a different notion of localisation called *strict localisability* [13,14]. The basic idea is that a state, localised within a region of space at some specific moment in time, should be such that the expectation value of any operator associated to a spacelike separated region should be the same as in the vacuum. In other words, average values of local operators will depend on the state only if the observation is made in the region where the state is localised. However, as shown by Knight, no finite superposition of N-particle states can be strictly localised. Some researchers have

[1] One way of seeing this is by noting that positive frequency solutions also satisfy the square root of the Klein–Gordon equation, i.e. the Schrödinger equation $i\dot{\phi}(\vec{x},t) = \sqrt{-\nabla^2+m^2}\phi(\vec{x},t)$. From there, using the *antilocality* property of the operator $\sqrt{-\nabla^2+m^2}$, it follows that the time derivative $\dot{\phi}$ is necessarily non-zero almost everywhere in space [6].