Annals of Physics 351 (2014) 200–249

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Excitation picture of an interacting Bose gas

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HIGHLIGHTS

- Excitation picture expresses interacting Bose gas with few atom clusters.
- Semiconductor and BEC many-body investigations are connected with cluster expansion.
- Quantum statistics of BEC is identified in terms of atom clusters.
- BEC number fluctuations show extreme sensitivity to many-body correlations.
- Cluster-expansion friendly framework is established for an interacting Bose gas.

ARTICLE INFO

Article history:
Received 21 July 2014
Accepted 30 August 2014
Available online 6 September 2014

Keywords:
Bose–Einstein condensate (BEC)
Strong many-body interactions
Cluster-expansion approach
Semiconductors vs. BEC
BEC quantum statistics

ABSTRACT

Atomic Bose–Einstein condensates (BECs) can be viewed as macroscopic objects where atoms form correlated atom clusters to all orders. Therefore, the presence of a BEC makes the direct use of the cluster-expansion approach – lucrative e.g. in semiconductor quantum optics – inefficient when solving the many-body kinetics of a strongly interacting Bose. An excitation picture is introduced with a nonunitary transformation that describes the system in terms of atom clusters within the normal component alone. The nontrivial properties of this transformation are systematically studied, which yields a cluster-expansion friendly formalism for a strongly interacting Bose gas. Its connections and corrections to the standard Hartree–Fock–Bogoliubov approach are discussed and the role of the order parameter and the Bogoliubov excitations are identified. The resulting interaction effects are shown to visibly modify number fluctuations of the BEC. Even when the BEC has a nearly perfect second-order coherence, the BEC number fluctuations can still resolve interaction-generated non-Poissonian fluctuations.

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http://dx.doi.org/10.1016/j.aop.2014.08.022

1. Introduction

The atomic Bose- and Fermi-gas investigations have become increasingly more ingenious ever since the discovery of atomic Bose–Einstein condensates (BECs) [1–3] in the mid 1990s. Nowadays, one can routinely confine multiple atomic clouds in free space [4,5] or on a lattice [6–8], and even make BECs interact with each other [5,9], or prepare a Fermi gas to exhibit quantum degeneracy [10–12], just to mention few highlights. At the same time, the development to control atom–atom interactions through a Feshbach resonance [13–15] has opened the possibility to systematically study [16–21] the many-body quantum kinetics of strongly interacting Bose/Fermi gas. Conceptually, these atomic investigations start to approach many-body problems that have been studied, e.g., in nonlinear semiconductor optics [22–34] for decades. Therefore, it clearly is interesting to explore which complementary insights many-body techniques – refined for the semiconductor studies – could provide for the strongly interacting Bose gas. In this paper, I develop a theoretic framework to connect these seemingly different many-body investigations, with the aim to identify the complementary aspects between typical semiconductor and BEC approaches.

Close to the equilibrium, an interacting Bose [35–39] or Fermi [14,40,41] gas can be accurately described with many sophisticated methods, which has provided detailed understanding of, e.g., many-body ground-state properties [42–44], BEC coherences [4,45–49], BEC dynamics [50–56] superfluidity [41,57–60], vortices [61–66], spectroscopic properties [67–71], so-called Tan relations [72–74] and their consequences [75–81], so-called BCS–BEC crossover [82–85], strong atom–atom interactions [7,19,86–92], and Efimov physics [93–98] in a strongly interacting atom gas. It also is interesting to study situations where the BEC is somehow excited far from the equilibrium, such as in the Bosenova experiments [99,100] where the BEC collapses due to a change in the atom–atom interactions. To explain the many-body quantum kinetics of the BEC, various perturbative approaches have been successfully used for weak interactions. One possibility is to apply the Hartree–Fock–Bogoliubov (HFB) [101–107] equations that couple the generalized Gross–Pitaevskii equation with the mean-field many-body dynamics of normal-component density and anomalous density. This approach qualitatively explains the spatial changes in the atom cloud during, e.g., the Bosenova implosion and eventual collapse of the atom cloud.

However, the HFB analysis cannot explain quantitatively the properties of BEC too far from equilibrium because it is based on the perturbation theory. For example, the HFB produces a collapse time that is up to 100% longer [107] than in the Bosenova experiment [99]. This analysis was extended in Ref. [108] to compare the HFB approach with the truncated Wigner approximation (TWA) [109–112] which produced essentially the same results; the outlook of this work concludes that one must extend both the TWA and the HFB approach to systematically include higher-order many-body correlations in order to quantitatively explain the nonperturbative phenomena such as the Bosenova.

The concepts of semiconductor quantum optics [32,33,113] could provide a complementary description for such correlations because they already provide an extremely accurate and nonperturbative nonequilibrium treatment [23,28,32,114,115] of the many-body and quantum-optical interaction effects [113,115–120] among fermionic electrons and bosonic photons [121–124] and phonons [125–127] far from equilibrium. When extending this approach for the strongly interacting Bose gas, one must first understand what happens when the atom–atom interactions become so strong that they can eject a large fraction of atoms from the BEC to the normal component. This process appears even at 0 K because the *interactions* among normal-component atoms may result to a lower energy than atoms have inside the BEC. This phenomenon is often referred to as *quantum depletion* [108,128,129] in contrast to thermal depletion of the BEC. The simplest description of such a process follows from Bogoliubov excitations, as experimentally demonstrated in Refs. [130–133] for a relatively weakly interacting Bose gas. As the interactions become stronger, significant modifications are expected based on the HFB insights discussed above.

My conceptualization of semiconductor quantum optics is founded on the general properties of the quantum statistics which is any representation defining uniquely *all* quantum properties of the many-body system, as formulated in Ref. [32,114]. For example, a density matrix or a Wigner function are possible choices for the quantum statistics. Alternatively, one may apply the cluster-expansion approach [28,32,114,134,135] to determine quantum statistics in terms of correlated