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Entanglement generation by electric field background

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ABSTRACT

The quantum vacuum is unstable under the influence of an external electric field and decays into pairs of charged particles, a process which is known as the Schwinger pair production. We propose and demonstrate that this electric field can generate entanglement. Using the Schwinger pair production for constant and pulsed electric fields, we study entanglement for scalar particles with zero spins and Dirac fermions. One can observe the variation of the entanglement produced for bosonic and fermionic modes with respect to different parameters.

1. Introduction

The quantum information theory is important for the multitude of its promising new applications in such varied fields as quantum communication and teleportation, quantum cryptography, quantum computing, etc. The concept of entanglement also plays crucial roles in black hole thermodynamics [1,2] and in the information loss problem [3–5], which have given rise to many studies aimed at measuring the generation and degradation of entanglement in a wide spectrum of systems. These studies include investigation of entanglement in both inertial [6] and non-inertial frames [7–12] as well as its generation in expanding spacetime [13,14] and in relativistic quantum fields [15].

Although many of these works are far from being experimental, they are valuable as they offer a refined understanding of quantum information. In this paper, we explore the generation of entanglement using Schwinger pair production. For this purpose, we will investigate the effect of background electric field on the generation of entanglement for scalar and spinor fields.

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It is well known that when an external electric field is applied to the quantum electrodynamical vacuum, the vacuum becomes unstable and decays into pairs of charged particles. In fact, the quantum vacuum is unstable under the influence of an external electric field, as the virtual electron–positron dipole pairs gain energy from the external field. When the field is sufficiently strong, these virtual pair particles gain the threshold pair creation energy and become real pairs. This remarkable phenomenon was first predicted by F. Sauter [16] to be later refined by W. Heisenberg and H. Euler [17] and formalized in the language of QED by Schwinger [18], hence its designation nowadays as the Schwinger pair production effect. This phenomenon has been investigated by scholars and workers from a variety of fields [19,20]. Efforts in the 1900s and early 21st century aimed at descriptions of more realistic field configurations led to the development of different formalisms such as the quantum kinetic approach which were used for the numerical computation of the Schwinger effect [21]. Other approaches used include the closely related scattering-like formalism in terms of the Riccati equation [22], the Dirac–Heisenberg–Wigner formalism [23], and the numerical worldline formalism [24]. The critical electric field required for pair creation is almost 10^{16} V/cm which is too enormous to be directly observed. However, the feasibility of its experimental realization in ultra-intense laser field system [25,26] has recently led to a re-thinking of the Schwinger effect. It has been realized that the Schwinger limit laser intensity of 4×10^{29} W/cm^2 is not necessarily a strict limit and might be lowered by several orders of magnitude through manipulating the form of the laser pulses [27–31]. Furthermore, it has been proposed that the Schwinger pair production effect may be observed in graphene [32]. These considerations motivated the authors to study the generation of entanglement using an electric field.

The present paper is organized as follows. In Section 2, we utilize the Schwinger effect for scalar particles with zero spin and Dirac fermions in the presence of a constant electric field. We will demonstrate that a constant electric field can generate the entanglement that its value can be determined. We will also consider the variation of the entanglement produced for bosonic and fermionic modes with respect to different parameters. In Section 3, we extend our investigation to the pulsed electric field. Finally, conclusions will be presented in Section 4.

2. Entanglement generation in a constant electric field

The Minkowski vacuum becomes unstable by a strong electric field and decays into pairs of charged particles. One can use the '*in*' and '*out*' formalism in order to investigate the entanglement generation. '*In*' and '*out*' are related to asymptotic times $t = -\infty$ and $t = +\infty$, respectively. If the separable '*in*' state can be expanded in terms of the entangled '*out*' state, the generated entanglement can then be determined. The state of two particles A and B is a vector in a $(d \times d')$-dimensional Hilbert space $H_{ab} = H_a \otimes H_b$. The space H_{ab} is the tensor product of the subspaces H_a and H_b of each particle. An element of the space H_{ab} is written as $|\Phi\rangle_{ab} = \sum_{i,j} C_{ij} |i\rangle_a \otimes |j\rangle_b$. A state $|\Phi\rangle_{ab} \in H_a \otimes H_b$ is separable if $|\Phi\rangle_{ab} = |\Phi\rangle_a \otimes |\Phi\rangle_b$. An entangled state is a state that is not separable [33].

In the following subsections, we study entanglement entropy for charged scalar and fermion particles in the presence of an electric field.

2.1. Entanglement entropy for scalar particles

In the study of entanglement generation, we use asymptotic solutions of equation of motion for charged scalar particles in the presence of an electric field.

Consider an electric field along the z-direction. It is related to the gauge potential through $E_z(t) = -\partial A_z(t)/\partial t$. For a scalar particle of mass m and Charge q, the Klein–Gordon equation on the four dimensional Minkowski spacetime with the metric $(+, -, -, -)$ is given by

$$[(\partial_\mu - iqA_\mu)(\partial^\mu - iqA^\mu) + m^2]\phi(t, x) = 0, \tag{1}$$

where, $A_\mu = (0, 0, 0, A_z(t))$ and ϕ is the scalar field.

For the purpose of the present subsection, we restrict ourselves to the constant electric field and rewrite Eq. (1) for $E_z(t) = E_0$:

$$[\partial_t^2 + m^2 + \hat{k}_\perp^2 + (\hat{k}_z - qE_0 t)^2]\phi_{ks}(t, x) = 0. \tag{2}$$