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Doorway states in the random-phase approximation

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ABSTRACT

By coupling a doorway state to a sea of random background states, we develop the theory of doorway states in the framework of the random-phase approximation (RPA). Because of the symmetry of the RPA equations, that theory is radically different from the standard description of doorway states in the shell model. We derive the Pastur equation in the limit of large matrix dimension and show that the results agree with those of matrix diagonalization in large spaces. The complexity of the Pastur equation does not allow for an analytical approach that would approximately describe the doorway state. Our numerical results display unexpected features: The coupling of the doorway state with states of opposite energy leads to strong mutual attraction.

1. Introduction

In the description of nuclear-structure phenomena, doorway states play an important role. Standard examples are the giant-dipole resonance [1] and, in medium-weight nuclei, low-lying isobaric analogue states [2]. A doorway state occurs when a distinct mode of nuclear excitation of given spin and parity, coupled strongly to the nuclear ground state or to some distinct scattering channel, is

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mixed with a background of states with the same quantum numbers. The strength of the mixing determines the spreading width of the ensuing resonance. For the giant-dipole resonance in even–even nuclei, the mode has spin/parity 1^- and is strongly coupled through the dipole operator to the nuclear ground state. The background states also have spin/parity 1^-. The resonance shows up in the cross section for photon absorption. The isobaric analogue state has isospin $T_0 + 1$ and is strongly coupled to the channel for scattering of protons on a target nucleus with one less proton. The background states have isospin T_0. The isobaric analogue resonance shows up in elastic proton scattering. Doorway states play an important role also in other areas of physics. By way of example we mention quantum information theory, mesoscopic physics, quantum chaos, and molecular physics. Without aiming at completeness, we refer to Refs. [3–10] and references therein.

In the standard theoretical description (see Ref. [11] and references therein), the doorway mode has energy E_0 and is coupled through real matrix elements V_μ, $\mu = 1, \ldots, N$ to N background states. These are governed by a real and symmetric Hamiltonian matrix $h_{\mu\nu}$ with $\mu, \nu = 1, \ldots, N$. In matrix form the total Hamiltonian H is given by

$$H = \begin{pmatrix} E_0 & V_\nu \\ V_\mu & h_{\mu\nu} \end{pmatrix}. \tag{1}$$

(For isobaric analogue resonances, Eq. (1) must be generalized to include the coupling of the analogue state with the background states via the proton channel, see Ref. [11].) Eq. (1) is patterned after the nuclear shell model. There, the dipole mode would be a linear superposition of one-particle one-hole states, the background states would be two-particle two-hole states, and E_0, the V_μ and the elements $h_{\mu\nu}$ would be determined by the single-particle energies and the residual interaction. For $N \gg 1$ a dynamical theory of the background states is not available in most cases, however, and the Hamiltonian matrix h is replaced by a matrix drawn at random from the Gaussian Orthogonal Ensemble of real symmetric matrices (the GOE). We have addressed the resulting problems in the theory of doorway states in two recent papers. In Ref. [12], we have worked out in a very general framework properties of doorway states as averages over the GOE in the limit $N \to \infty$. Properties of the spreading width that emerge beyond the standard approximation were investigated in Ref. [13]. An essential and generic feature of the doorway state model is that the value of the spreading width is adjustable. Moreover, this value is of order $1/N$ in relation to the overall width of the spectrum of the background states. This last property guarantees that the doorway state is a local spectral phenomenon.

In the present paper we extend the concept and the description of doorway states to the random-phase approximation (RPA). Our extension is motivated by the fact that in nuclear-structure theory, it is often mandatory to replace the shell-model approach embodied in Eq. (1) by the RPA [14]. That is true especially for the treatment of collective motion. The RPA is characterized by symmetries that are radically different from those of the Hamiltonian approach in Eq. (1). Our extension takes account of these symmetries. Specifically, it involves four elements. (i) We need an RPA model for the doorway state as a collective state. (ii) Similar to the replacement of $h_{\mu\nu}$ by the GOE, our RPA model must involve a random-matrix model with RPA symmetries for the background states. (iii) The coupling of the doorway state to the background states (the analogue of the matrix elements V_μ) must also possess RPA symmetries. (iv) The value of the spreading width due to that coupling must be an adjustable parameter, and it must be of order $1/N$ in relation to the overall width of the spectrum of the background states. The resulting theory of the doorway phenomenon in RPA turns out to be radically different from the standard approach.

The random-matrix approach to RPA equations has been formulated and investigated in some detail in Ref. [15]. In that paper a follow-up paper was announced that would combine the purely statistical (or "democratic") description of the background states in terms of a random-matrix model with the highly special dynamical RPA description of a select state, the doorway state. Aside from being an extension of our investigation of the doorway state phenomenon in Refs. [12,13], the present paper may also be viewed as that follow-up paper. It might, therefore, also carry the title "Random-Matrix Approach to RPA Equations II". In the paper we are mainly interested in the consequences the RPA symmetry has for the doorway state picture. We do not discuss any applications.