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Multikink solutions and deformed defects

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ABSTRACT

In this work we consider an application of the deformation procedure that enables us to construct, systematically, scalar field models supporting multikinks. We introduce a new deformation function in order to realize this task. We exemplify the procedure with three different starting models already known in the literature, and the resulting deformed models have rich minima structures which are responsible for the appearance of multi-kink configurations. We have also considered an application to braneworld scenarios, where we have obtained interesting configurations corresponding to the multikink solutions.

1. Introduction

In the last few decades, defects structures have received considerable amount of attention in the literature. In fact, topological defects play an important role in the development in several branches of physics, from condensed matter to high energy physics and cosmology [1–4]. In field theory they usually emerge from models supporting spontaneous symmetry breaking, and they may appear as kinks, domain walls, vortices, strings and monopoles.

In condensed matter, a recent and interesting example regarding topological defects is related with the study of magnetic domain wall in a nanowire, designed for the development of magnetic memory [5]. Also in the context of condensed matter, in Refs. [6–8] was shown that the presence of kink-like defects in quasi-one-dimensional systems like polyacetylene is important for the increase of conductivity to almost metallic level of this insulator, when it introduced charged defects by doping. In high energy physics we may cite, for instance, the importance of defect structures in brane world

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scenarios, where we may interpret that we live in a domain-wall with $3+1$ dimensions embedded in a 5-dimensional spacetime [9]. In the context of braneworlds with warped geometry, kink-like defects are used to engender the 5-dimensional spacetime structure [10]. In cosmology, topological defects may be related with phase transitions in the early Universe, such defects may have been formed as the Universe cooled and various local and global symmetries were broken [2,11].

Some time ago, Peyrard and Kruskal [12] discovered that a single kink becomes unstable when it moves in a discrete lattice with large velocities, while multikink solutions remain stable. This effect is associated with the interaction between the kink and the radiation, and the resonances were already observed experimentally [13]. The above reasons motivated the study of multikinks and, some years ago, Champneys and Kivshar [14] performed an analysis on the reasons of the appearance of multikinks in dispersive nonlinear systems. Furthermore, multikinks have applications, for instance, in the study of mobility hysteresis in a damped driven commensurable chain of atoms [15]. Moreover, in arrays of Josephson junctions, instabilities of fast kinks generate bunched fluxon states presenting multikink profiles [16]. However, at the present state there is a few number of scalar field models that present multikink solutions. Recently, we proposed a model with smooth potential which supports multikink configurations [17], however, in that case the "smoothness" of the potential is guaranteed only up to the first derivative. In another paper, we proposed a model for doublets of scalar field where one of its components is a special case of multikinks, the triple-kink [18].

Since kink-like configurations are obtained from nonlinear field equations, it is difficult to obtain analytical solutions and, as a consequence, any method that helps us in this task would be certainly welcomed. Some years ago, Bazeia and collaborators introduced the so-called deformation procedure [19], which enables us to construct new scalar field models from a starting one. This procedure was wisely applied to a large number of scalar field models, see [20] and references therein.

In this paper we will consider an application of the deformation procedure that enables us to construct, in a systematic way, new scalar field models which support multikink solutions. In order to realize this task, we introduce a new deformation function. The procedure will be exemplified with three different starting models that are well known in the literature, and the resulting models possess rich minima structures which are responsible for the appearance of multikink configurations.

This work is organized as follows: in Section 2 we review the basic ideas on the deformation procedure that will be necessary for this work. In Section 3 we introduce the deformation function that will be used in this paper and consider three explicit examples of the generation of multikinks. In Section 4 we discuss the stability against small fluctuations. In Section 5, we consider an application of one of the models presented here to braneworld scenarios. Finally, in Section 6 we conclude.

2. Deformation procedure

Some years ago Bazeia and collaborators [19] introduced a procedure that enables us to construct new models supporting topological defects from a starting model. In this section we will review the main aspects on the deformation procedure. Let us consider two models of real scalar field in $1+1$ dimensions described by the respective Lagrangian densities

$$\mathcal{L}_j = \frac{1}{2}\partial_\mu\phi_j\partial^\mu\phi_j - V_j(\phi_j), \qquad \mathcal{L}_i = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V_i(\phi_i). \tag{1}$$

The first order equations for the static solutions of minimal energy configuration are given by

$$\frac{1}{2}\left(\frac{d\phi_j}{dx}\right)^2 = V_j(\phi_j) \quad \text{and} \quad \frac{1}{2}\left(\frac{d\phi_i}{dx}\right)^2 = V_i(\phi_i). \tag{2}$$

It is possible to introduce a function $\phi_j = f_{ji}(\phi_i)$, called deformation function, that connects the model described by $\mathcal{L}_i$ to the model $\mathcal{L}_j$ by relating the potentials $V_i(\phi_i)$ and $V_j(\phi_j)$ in the very specific form

$$V_i(\phi_i) = \frac{V_j(f_{ji}(\phi_i))}{f'_{ji}(\phi_i)^2}, \tag{3}$$

where the prime denotes the derivative with respect to the argument of the function. This procedure allows us to start with a real scalar field model described by the Lagrangian density $\mathcal{L}_j$ whose