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Dissipative Bohmian mechanics within the Caldirola–Kanai framework: A trajectory analysis of wave-packet dynamics in viscid media

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HIGHLIGHTS

- A dissipative Bohmian approach is developed within the Caldirola–Kanai model.
- Some simple yet physically insightful systems are then studied analytically.
- Dissipation leads to spatial localization in free-force regimes.
- Under the action of linear forces, dissipation leads to uniform motion.
- In harmonic potentials, the system decays unavoidable to the well minimum.

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ABSTRACT

Classical viscid media are quite common in our everyday life. However, we are not used to find such media in quantum mechanics, and much less to analyze their effects on the dynamics of quantum systems. In this regard, the Caldirola–Kanai time-dependent Hamiltonian constitutes an appealing model, accounting for friction without including environmental fluctuations (as it happens, for example, with quantum Brownian motion). Here, a Bohmian analysis of the associated friction dynamics is provided in order

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Quantum fluid dynamics
Quantum dissipation
Open quantum system

to understand how a hypothetical, purely quantum viscid medium would act on a wave packet from a (quantum) hydrodynamic viewpoint. To this purpose, a series of paradigmatic contexts have been chosen, such as the free particle, the motion under the action of a linear potential, the harmonic oscillator, or the superposition of two coherent wave packets. Apart from their analyticity, these examples illustrate interesting emerging behaviors, such as localization by "quantum freezing" or a particular type of quantum–classical correspondence. The reliability of the results analytically determined has been checked by means of numerical simulations, which has served to investigate other problems lacking of such analyticity (e.g., the coherent superpositions).

1. Introduction

An iron ball falling in oil reaches after some time a constant velocity. It is then not accelerated anymore. The same happens with rain drops in air. These are two examples of our everyday life where a classical system undergoes a uniform motion when acted by a viscid medium (oil in the first example and air in the second one). This effect arises when the friction with such a medium compensates the acceleration induced by the gravity acting on the system. But, what about quantum systems? May they display such behaviors? Actually, how would a viscid quantum medium act on a quantum system, if such a medium exists? These are natural questions that come to our minds when trying to establish a correspondence with the analogous classical systems. However, before considering a model to describe such a situation, it is interesting to make some considerations on quantum systems in the light of the theory of open quantum systems [1,2].

Real systems are not in isolation in Nature. Rather, we find them coupled to other systems, namely an environment or bath. The usual way to tackle their study is first by considering a system-plus-reservoir Hamiltonian system, which includes all the involved degrees of freedom (those from the system plus those from the environment) as well as their coupling. However, not always a full quantum-mechanical description in these terms is affordable and therefore one has to consider simpler models consisting of the bare (isolated) system Hamiltonian plus an effective, time-dependent interaction. Although these models do not provide us with any information regarding the environmental dynamics, they are very convenient to describe its effects on the system, even if the problem becomes *nonconservative*. Phenomenological equations only accounting for the system dynamics are, for example, the Lindblad or the quantum Langevin equations. In the first case, an effective description of the evolution of the (reduced) system density matrix is achieved by including a series of dissipation operators or dissipators in the corresponding equation of motion. In the second case, the equation describes the evolution of an operator associated with a certain observable (e.g., the system position), and apart from a dissipative term the equation also includes a stochastic noise related to the environmental fluctuations. Nevertheless, in both cases the effect on the system dynamics is the same: a loss of the system coherence (decoherence) and a relaxation or damping of its energy.

In the particular case of the quantum Langevin equation, a (quantum) noise function is included in order to account for the environmental thermal fluctuations on the system (Brownian-like motion). This equation can be reformulated in terms of a Hamiltonian model, where the noise arises from a collection of harmonic oscillators coupled to the system. This is the so-called Caldeira–Leggett model [3,4]. Predating this model, though, we find a former dissipative one, the so-called Caldirola–Kanai model [5,6]. This is a Hamiltonian reformulation of the Langevin equation with zero fluctuations, i.e., the Brownian-like thermal fluctuations that sustain the system motion are neglected and the