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$\mathbb{Z}_N$ symmetric chiral Rabi model: A new N-level system

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ABSTRACT

We present a new tractable quantum Rabi model for N-level atoms by extending the $\mathbb{Z}_2$ symmetry of the two-state Rabi model. The Hamiltonian is $\mathbb{Z}_N$ symmetric and allows the parameters in the level separation terms to be complex while remaining hermitian. This latter property means that the new model is *chiral*, which makes it differ from any existing N-state Rabi models in the literature. The $\mathbb{Z}_N$ symmetry provides partial diagonalization of the general Hamiltonian. The exact isolated (i.e. exceptional) energies of the model have the Rabi-like form but are N-fold degenerate. For the three-state case ($N = 3$), we obtain three transcendental functions whose zeros give the regular (i.e. non-exceptional) energies of the model.

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1. Introduction

Matter–light interactions are ubiquitous in nature. In modern physics, they are modeled by systems of atoms interacting with boson modes (i.e. spins coupled to harmonic oscillators). One of the best-known spin-boson systems is the phenomenological quantum Rabi model [1–3] which continues to be a subject of significant interest [4–12]. This model describes the interaction of a two-level atom with a cavity mode of quantized electromagnetic field, i.e. a single spin-1/2 particle coupled to a harmonic oscillator. Due to the simplicity of its Hamiltonian, the Rabi model has served as the theoretical basis for understanding the interactions between matter and light, and has found a variety of applications ranging from quantum optics [13], solid state semiconductor systems [14], molecular physics [15] to quantum information [16]. With experimental techniques now available to access ultra-strong atom–cavity coupling regimes [17], there is also much ongoing interest in experimental realizations of the Rabi interactions in both circuit and cavity quantum electrodynamics (QED) [18–24].

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The quantum Rabi model is the simplest spin-boson system without the rotating wave approximation (RWA). To understand more sophisticated spin-boson interactions, e.g. multi-state atom–cavity interaction [25–27], there is a need for extensions of the two-level Rabi model. In this regard, let us mention three research directions which have attracted significant attention. One is the study of the Dicke model which couples N two-level systems to a single radiation mode and is relevant to experimental realization and applications in quantum computing. There is a vast literature on the Dicke model. Analytic solutions for the $N = 2, 3$ Dicke models have recently been studied in [28–30]. Another one concerns models of a two-level atom interacting with multi harmonic modes or with a higher-order harmonic generation. Examples include the two-mode [31] and 2-photon [32] Rabi models recently solved analytically in [11,33]. These models can be experimentally realized in circuit QED systems [19] and have established applications in e.g. Rubidium atoms [34] and quantum dots [35,36]. The third direction is to consider multi-state atom–cavity interactions, i.e. multi-level atoms coupled to harmonic boson modes. Early works on this aspect include [37,3]. The former discussed a very special N-level system interacting with a boson mode and the latter studied generalized Fulton–Gouterman transformation for systems with abelian symmetry including certain $\mathbb{Z}_N$-invariant systems. Most previous multi-level extensions [38–42] are neither tractable nor applicable to atom–cavity systems. Recently the author in [43] proposed a tractable quantum Rabi model for N-state atoms.

In this paper we introduce a different, tractable quantum Rabi model for N-level atoms. The Hamiltonian of the new model is $\mathbb{Z}_N$ symmetric, extending the $\mathbb{Z}_2$ symmetry of the two-state model. One of the unique features to our model is that it allows the parameters in the level-splitting terms (α_m in (2) below) to be complex while keeping the Hamiltonian hermitian. It is therefore a *chiral* system [44] and is referred to as $\mathbb{Z}_N$ symmetric N-state chiral Rabi model. The $\mathbb{Z}_N$ symmetry provides a partial diagonalization of the Hamiltonian. It is found that the exact isolated (i.e. exceptional) energies of the model have the Rabi-like form but are N-fold degenerate. They correspond to polynomial solutions of the Schrödinger equation and appear when the model parameters satisfy certain constraints. For the three-state case ($N = 3$), we analytically determine three transcendental functions whose zeros give the regular energies of the system. Our results pave the way for applications to multi-level atom–cavity experiments.

2. $\mathbb{Z}_N$ symmetric Rabi Hamiltonian

The two-state Rabi Hamiltonian is $\mathbb{Z}_2$ symmetric. So the most natural N-state generalization of the Rabi model would be given by a Hamiltonian with $\mathbb{Z}_N$ symmetry. We can proceed in the following intuitive and mathematically rigorous way. Similar to the Rabi case where a two-level atom is modeled by spins with two states (Pauli matrices σ_z and σ_x), we model an N-level atom by "spins" with N states. Then the Hilbert space of the N-state atom is the N-dimensional vector space $\mathbb{C}^N$. Let Z and X be the basic operators which generalize the Pauli matrices σ_z and σ_x to $\mathbb{C}^N$, respectively. Instead of anti-commutation relations, these operators satisfy [45]

$$\begin{aligned} &Z^N = X^N = 1, \qquad Z^\dagger = Z^{N-1}, \qquad X^\dagger = X^{N-1}, \\ &ZX = \omega XZ, \quad \omega = e^{2\pi i/N}. \end{aligned} \tag{1}$$

It is useful to keep in mind some explicit representations of these operators. Diagonalizing the operator Z gives $Z = \mathrm{diag}(1, \omega, \omega^2, \ldots, \omega^{N-1})$ and $X_{l,m} = \delta_{l,m+1\,(\mathrm{mod}\,N)}$. On the other hand, from (1) we have $X^\dagger Z = \omega Z X^\dagger$. Thus the representation in which X is diagonal is given by $X^\dagger = \mathrm{diag}(1, \omega, \omega^2, \ldots, \omega^{N-1})$ and $Z_{l,m} = \delta_{l,m+1\,(\mathrm{mod}\,N)}$. This latter representation is useful in what follows.

Then the most natural and mathematically manageable N-state generalization of the two-state Rabi model can be obtained by replacing the Pauli matrices in the latter model with the "spins" with N states. It is convenient to label the N-states by $1,\ \omega,\ \omega^2,\ \ldots,\ \omega^{N-1}$. We thus arrive at the following Hamiltonian of a N-level atom interacting with a boson mode,

$$H_N = \Omega b^\dagger b + \Delta \sum_{m=1}^{N-1} \alpha_m Z^m + \lambda \left(X^\dagger b^\dagger + X b \right), \tag{2}$$

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