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Mean-field description of pairing effects, BKT physics, and superfluidity in 2D Bose gases

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HIGHLIGHTS

- We developed a mean-field theory for 2D bosons with intermediate interaction strength at finite temperature.
- Renormalized coupling constant shows agreement with t-matrix calculations.
- We incorporate BKT physics by considering phase fluctuations.
- Finite pairing amplitude may survive above the BKT transition temperature.
- We found a two-peak structure in the spectral function indicating the finite pairing amplitude.

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ABSTRACT

We derive a mean-field description for two-dimensional (2D) interacting Bose gases at arbitrary temperatures. We find that genuine Bose–Einstein condensation with long-range coherence only survives at zero temperature. At finite temperatures, many-body pairing effects included in our mean-field theory introduce a finite amplitude for the pairing density, which results in a finite superfluid density. We incorporate Berezinskii–Kosterlitz–Thouless (BKT) physics into our model by considering the phase fluctuations of our pairing field. This then leads to the result that the superfluid phase is only stable below the BKT temperature due to these phase fluctuations. In the weakly interacting regime at low temperature we compare our theory to previous results from perturbative calculations, renormalization group calculations as well as Monte Carlo simulations. We present a finite-temperature phase diagram of 2D Bose gases. One signature of the finite amplitude of the pairing

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density field is a two-peak structure in the single-particle spectral function, resembling that of the pseudogap phase in 2D attractive Fermi gases.

1. Introduction

Recent experiments on two-dimensional (2D) ultra-cold atoms have explored many interesting phenomena including the Berezinskii–Kosterlitz–Thouless (BKT) physics [1], superfluidity [2], scale invariance [3], radio-frequency (RF) spectroscopy [4], thermodynamics [5], pseudogap physics above the BKT transition temperature [6], and others. These experiments provide opportunities for studying more complicated 2D or layered systems related to high-temperature superconductors [7,8] and interface superconductivity [9]. Theoretical studies on dilute 2D Bose gases have been reviewed in Ref. [10] and those available theories are confined to weakly interacting regimes or temperatures close to zero or near the critical regime. As a consequence, finite-temperature phase diagrams shown in Ref. [10] are schematic instead of coming from a consistent theoretical description. Ref. [11] discusses other many-body techniques for addressing 2D bosons and their potential issues. A mean-field theory that works in the regime of intermediate interaction strength at arbitrary temperature thus would be highly desired for a systematic analysis of 2D interacting Bose gases. To better understand the physics, one needs a coherent description of superfluidity, the BKT transition, pairing effects, and single-particle excitation energy. The goal of this paper is to present a plausible mean-field theory with experimental consequences for a 2D interacting single-species Bose gas.

For an attractive 2D two-component Fermi gas, there have been theories based on the phase fluctuations of the BCS theory and its extension to the Bose–Einstein condensation (BEC) of dimers [7,12]. When the temperature T is below a pairing onset temperature, pairs with disordered phases emerge. When T falls below the BKT transition temperature T_{BKT}, a superfluid phase becomes stable but a genuine long-range ordered phase only survives at $T = 0$. For bosons we may explore similar physics. Several questions follow: How can BKT physics be incorporated into a theory of 2D interacting bosons? Does any interesting phase exist above T_{BKT}? Can 2D bosons have an energy gap in the single-particle excitation? These issues will be addressed in a consistent theoretical framework.

Ref. [10] poses a series of questions that needs to be explored in the theoretical work on 2D interacting Bose gases. The last one is "Could one justify a large-N approach which improves on existing methods by incorporating the t-matrix approximation?". Inspired by this question, here we base our theory on the leading-order-auxiliary-field (LOAF) theory of interacting bosons [13,14], which is a generalization of the conventional large-N expansion [15–17]. It also reduces to the large-N expansion in the normal phase. The LOAF theory is a mean-field theory for interacting Bose gases. Its advantages in describing a 3D Bose gas beyond perturbative regimes at arbitrary temperature are summarized below. In the following we will construct a mean-field theory that applies to 2D Bose gases and explore its thermodynamics and possible experimental implications.

For 3D interacting bosons the LOAF theory meets three important criteria by treating the pairing (anomalous) density field and the (normal) density field on an equal footing: (i) a gapless dispersion in the BEC phase, (ii) a conserving theory, and (iii) prediction of a second-order BEC transition. Widely used theories such as the Hartree–Fock theory or the Popov theory fail at least one criterion [14]. Moreover, the LOAF theory exhibits a shift in the critical temperature T_c consistent with the results of Ref. [18]. We emphasize that the LOAF theory naturally recovers the Bogoliubov theory of weakly interacting bosons [13] and note that two Green's functions corresponding to our two density fields are indeed present in the Bogoliubov theory [19]. An important feature of the LOAF approximation is that the superfluid density is closely related to the pairing density [20] and the two quantities obey a Josephson relation [21]. This will be crucial in integrating the BKT physics into the LOAF theory of 2D Bose gases.