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Hamiltonian dynamics of an exotic action for gravity in three dimensions

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HIGHLIGHTS

- We report a detailed Hamiltonian analysis for an exotic action of gravity.
- We show that Palatini and exotic actions are not equivalent.
- The exotic action is a non-commutative theory.
- The fundamental gauge transformations of the theory are Λ-deformed Poincaré transformations.
- A Lorentz and gauge invariant symplectic two-form is constructed.

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ABSTRACT

The Hamiltonian dynamics and the canonical covariant formalism for an exotic action in three dimensions are performed. By working with the complete phase space, we report a complete Hamiltonian description of the theory such as the extended action, the extended Hamiltonian, the algebra among the constraints, the Dirac's brackets and the correct gauge transformations. In addition, we show that in spite of exotic action and tetrad gravity with a cosmological constant give rise to the same equations of motion, they are not equivalent, in fact, we show that their corresponding Dirac's brackets are quite different. Finally, we construct a gauge invariant symplectic form which in turn represents a complete Hamiltonian description of the covariant phase space.

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1. Introduction

A dynamical system is characterized by means of its symmetries which constitute an important information in both the classical and the quantum context. It is well-known that the analysis of a dynamical system by means of its equations of motion implies that the phase space is not endowed

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with a natural or preferred symplectic structure as it has been claimed in [1,2], and the freedom in the choice of the symplectic structure is an important issue because it could yield different quantum formulations. Hence, in spite we have an infinite way to choose a symplectic structure for any system, the following question arises: are there the same symmetries in two different actions sharing the same equations of motion? The answer in general is not. In fact, it has been showed that two theories sharing the same equations of motion, does not imply that the theories are equivalent even at the classical level [3,4]. Nonetheless, the study of any theory should be carried out extending the definition of a dynamical system by considering its equations of motion plus an action principle, thus we are in a profitable situation because the action gives us the equations of motion and symmetries; additionally it fixes the symplectic structure of the theory [2,5]. In this manner, in the study of the symmetries of a dynamical system must be taken into account both, the equations of motion plus an action principle [4]. Nowadays, there exist approaches that can be used for studying the symmetries of any theory, as for instance, Dirac's canonical formalism and the covariant canonical method, both with their respective advantages. Dirac's canonical formalism is an elegant approach for obtaining relevant physical information of a theory under study, namely, the counting of physical degrees of freedom, the correct gauge transformations, the study of the constraints, the extended Hamiltonian and the extended action [6], all this information is the guideline to make the best progress in the analysis of quantum aspects. On the other hand, in the covariant canonical method, in order to describe all the relevant Hamiltonian description of the covariant phase space [7], we are able to identify a gauge invariant two-form, being an important step to analyze within a complete covariant context the theory under study. Therefore, we think that the complete analysis of any theory should be done by performing a Dirac's canonical approach and the canonical covariant method, the former because it considers the action to study its symmetries, the latter takes into account the equations of motion in order to construct the covariant phase space. In this respect, usually the way to perform the Dirac formalism is not carried out in a complete form, namely, usually the people prefer to work on a smaller phase space context [8–10]; this means that only those variables that occur in the action with temporal derivative are considered as dynamical, in general in order to obtain a complete study one must perform a pure Dirac's method, this is, we need to consider the complete set of variables occurring in our theory as dynamical ones. In this respect, we have performed a pure Dirac's canonical analysis for models as *BF* theories, the Pontryagin invariant, topological theories, etc., [9,10] and we have reported the complete structure of the constraints defined on the full phase space, we have commented in those works, that by performing a pure Dirac's framework we are able to know the symmetries of the theory, as for instance, gauge symmetry and the complete algebra among the constraints defined on the full phase space, fact that usually is not possible to obtain by using a smaller phase space context.

In this manner, the purpose of this paper, is to develop a complete Hamiltonian analysis of an exotic action in three dimensions. It is well-known that Palatini's gravity with a cosmological constant and exotic action yield the same equations of motion, and there are many works commenting that this fact makes the actions classically equivalent (see [11,12] and the references therein). However, a complete analysis of an exotic action has not been performed, and therefore the complete symmetries of the theory are not well known. Thus, we show in this paper that the Dirac's brackets for the dynamical variables that define exotic action and Palatini's gravity with a cosmological constant are different. In fact, for the former the dynamical variables of the theory are non-commutative and the cosmological constant cannot be zero. For the Palatini action with a cosmological constant, the Dirac's brackets of dynamical variables are commutative and the cosmological constant can be taken as zero, all those ideas will be clarified along the paper. In addition we report the canonical covariant analysis of an exotic action in order to report a complete study of the theory. By constructing a gauge invariant two form on the covariant phase space, we confirm some results obtained by means of Dirac's framework.

2. Hamiltonian dynamics for an exotic action in three dimensions

In this section, we will perform a pure Dirac's analysis for an exotic action given by the following action [12]

$$S[e,A]_{\text{exotic}} = \frac{1}{2}\int_M A^{IJ}\wedge dA_{IJ} + \frac{2}{3}A^{IK}\wedge A_{KL}\wedge A^{L}{}_{I} + \int_M \frac{\Lambda}{2} e_I \wedge De^I, \tag{1}$$

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