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Supersymmetric descendants of self-adjointly extended quantum mechanical Hamiltonians

M.H. Al-Hashimi [a,*], M. Salman [b,1], A. Shalaby [b,c,2], U.-J. Wiese [a,d,3]

[a] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, Bern University, Sidlerstrasse 5, CH-3012 Bern, Switzerland*

[b] *Department of Mathematics, Statistics, and Physics, Qatar University, Al Tarfa, Doha 2713, Qatar*

[c] *Physics Department, Faculty of Science, Mansoura University, Egypt*

[d] *Center for Theoretical Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA, USA*

HIGHLIGHTS

- Self-adjoint extension theory and contact interactions.
- Application of self-adjoint extensions to supersymmetry.
- Contact interactions in finite volume with Robin boundary condition.

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ABSTRACT

We consider the descendants of self-adjointly extended Hamiltonians in supersymmetric quantum mechanics on a half-line, on an interval, and on a punctured line or interval. While there is a 4-parameter family of self-adjointly extended Hamiltonians on a punctured line, only a 3-parameter sub-family has supersymmetric descendants that are themselves self-adjoint. We also address the self-adjointness of an operator related to the supercharge, and point out that only a sub-class of its most general self-adjoint extensions is physical. Besides a general characterization of self-adjoint extensions and their supersymmetric descendants, we

* Corresponding author. Tel.: +41 31 631 8878.
E-mail addresses: hashimi@itp.unibe.ch (M.H. Al-Hashimi), msalman@qu.edu.qa (M. Salman), amshalab@qu.edu.qa (A. Shalaby), wiese@itp.unibe.ch (U.-J. Wiese).

1 Tel.: +974 4403 4618.

2 Tel.: +974 4403 4630.

3 Tel.: +41 31 613 8504.

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explicitly consider concrete examples, including a particle in a box with general boundary conditions, with and without an additional point interaction. We also discuss bulk-boundary resonances and their manifestation in the supersymmetric descendant.
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1. Introduction

The differences between Hermiticity and self-adjointness of quantum mechanical operators [1] were first understood by von Neumann [2], but are rarely emphasized in the quantum mechanics textbook literature. Even standard textbook problems such as a particle confined to a box [3] or endowed with a point interaction [4–7] become much richer when studied systematically in the context of self-adjointly extended Hamiltonians [8]. Self-adjoint extensions arise naturally at spatial boundaries, such as the interfaces in semiconductor heterostructures including quantum dots, quantum wires, and quantum wells [9], or at singular points, e.g. at the location of a cosmic string or vortex, which may manifest themselves as the tip of a cone in $(2 + 1)$ space–time dimensions [10–13]. At a spatial boundary, a real-valued self-adjoint extension parameter characterizes the so-called Robin boundary conditions, which interpolate between Dirichlet and Neumann boundary conditions [14–19]. Robin boundary conditions have also been used in quantum field theory, for example, in investigations of the Casimir effect [20,21] and of the AdS/CFT correspondence [22]. The confinement of atoms or molecules in a finite region of space is an important subject in nanotechnology [23]. Recently, we have derived a generalized Heisenberg uncertainty relation for a quantum dot with general Robin boundary conditions [8]. As special cases, we have investigated electrons in a spherical cavity bound to its center by harmonic [24] or Coulomb forces [25], with a focus on the resulting accidental symmetries [26,27]. General Robin boundary conditions may lead to bound states localized on the confining wall. In these cases, we have encountered bulk-boundary resonances whose wave functions are partly localized near the boundary and partly near the center of the cavity.

Supersymmetric quantum mechanics has deepened the understanding of quantum mechanics by associating a chain of supersymmetric descendants to a given quantum mechanical Hamiltonian [28–31]. In this way, the number of analytically solvable quantum mechanical problems has been extended significantly. General point interactions have been investigated in the framework of supersymmetric quantum mechanics in [32–37]. Here we study the superpartners of self-adjointly extended Hamiltonians. Remarkably, these are not automatically self-adjoint. In particular, only a 3-parameter sub-family of the general 4-parameter family of self-adjointly extended Hamiltonians on a punctured line have supersymmetric descendants that are themselves self-adjoint. We also construct the self-adjoint extensions of an operator related to the supercharge. Since we consider a Hamiltonian and its supersymmetric descendant as two different physical systems, rather than as two parts of the same system, only a sub-class of self-adjoint extensions of this operator is physical.

We illustrate our general results with specific systems, including a particle confined to a box with or without an additional point interaction. We will again encounter bulk-boundary resonances, and we study their manifestation in the corresponding supersymmetric descendant. While there are experimental realizations of bulk-boundary resonances, e.g. for atoms encapsulated in fullerenes [38,39], our current study is not motivated by a particular application. Instead, we aim at illuminating the relations between the theoretical concepts of supersymmetry and self-adjoint extensions in quantum mechanics in general. Self-adjoint extensions of specific supersymmetric Hamiltonians have also been considered in the context of the Aharonov–Bohm effect [40–42].

Some of the concrete systems studied here are so simple that they could easily serve as problems in the teaching of quantum mechanics. In this sense, our paper also has pedagogical intentions, by trying to convince the reader that the theory of self-adjoint extensions is not only mathematically elegant, but also of great physical relevance, that deserves a more prominent place in the teaching of quantum mechanics.

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