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# Modified mean-field formulations for the improved simulation of short fiber reinforced thermoplastics

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ABSTRACT

The aim of the contribution is to introduce modified mean-field formulations for the improved simulation of short fiber reinforced thermoplastics. In the first part, the recently proposed second moment incremental formulation for the mean-field homogenization of elastic–plastic composites of Doghri et al. (2011) [1] is modified. A stress concentration factor is introduced, which enables an adequate consideration of stress and strain inhomogeneities in dependence of the fiber orientation. In the second part, special focus is put on the characteristic elastic–plastic behavior of thermoplastics. It is well known, that thermoplastics show a distinct dependence of the volumetric stress in their mechanical elastic–plastic behavior. To account for this dependency, the commonly in mean-field formulations integrated von-Mises plasticity model is replaced by a quadric yield formulation proposed by Kolling et al. (2005) [2]. Conclusive results are achieved with both formulations by comparing simulation and experimental results.



## 1. Introduction and motivation

Mean-field homogenization formulations are efficient micromechanical methods, which enable simulations of composite parts and structures at reasonable computational cost. For the problem of a single inclusion embedded in an infinite matrix, results are achievable by applying the solution proposed by Eshelby [3] for the partitioning of strains and stresses among the phases. Additional assumptions are necessary to consider short fiber reinforced thermoplastics and interactions between inclusions. The Mori–Tanaka mean-field homogenization scheme of Mori and Tanaka [4], which was further developed by Benveniste [5] and Tandon and Weng [6] allows the consideration of these interactions in an average way. Tucker and Liang [7] showed that, as long as a linear elastic material behavior is considered, good results are achievable. The extension of these methods for the computation of the elastic–plastic mechanical behavior of fiber reinforced composites commonly rely on the incremental approach proposed by Hill [8]. This approach was further developed for example by Pettermann et al. [9]. Later on, Doghri and Ouaar [10] improved its predictive capabilities by considering for example only the isotropic part of the tangent operator for the computation of the Eshelby's tensor. Pierard and Doghri [11] study various estimates for the computation of a tangent operator, for example by analyzing the influence of different isotropization methods and their influence on the overall mechanical composite behavior. Doghri et al. [12] invested an enormous effort to understand and improve the predictive capabilities of mean-field homogenization methods, for example by introducing an general incrementally affine linearization method for an elasto–viscoplastic matrix behavior. Another promising direction towards the development of advanced and rigorous homogenization models is provided by incremental variational principles. For instance, incremental variational estimates were proposed by Lahellec and Suquet [13,14] in the context of nonlinear viscoelasticity. Recently, Brassart et al. [15] proposed a new mean-field homogenization model based on an incremental variational principle for elasto–viscoplastic composites. The new model is an extension of the previous formulation for rate-independent elasto-plasticity (Brassart et al. [16]). However, these modeling strategies are not the focus of this contribution and the interested reader my find more information by studying in references mentioned above.

Besides the above mentioned improvements by considering an incremental Mori–Tanaka (iMT) formulation two challenges remain unsettled and are discussed in detail in the following subsections. Such a formulation tends to overestimate the yield point in fiber direction. This is shown for example by Pierard et al. [17]. The authors compare the effective mechanical behavior of an elasto-plastic matrix reinforced with elastic fibers. The results are obtained by conducting finite element simulations of an iMT and

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of a representative composite model (RCM). In a RCM, which is a kind of a representative volume element, several fibers are modeled and embedded randomly into a matrix by using finite elements and by applying periodic boundary conditions. In this contribution such simulations are also applied to evaluate the existing and the proposed modeling approaches. The effect of overestimating the yield point is shown in Fig. 1. The details of the applied iMT approach and the applied isotropization method can be found in Doghri and Tinel [18], Doghri and Tinel [19]. The considered material in this contribution is a polybutylene terephthalate filled with 20% glass fiber (PBT-GF, Celanex 2300 GV1-20, Ticona). The material data is summarized in Table 1. In the first finite element simulations, a classical von-Mises plasticity model is used for modeling the elastic–plastic behavior of the matrix. Hence, the yield function can be defined by:

$$f^{\text{Mises}} = \sigma_{eq} - R(\varepsilon^{pl}) \tag{1.1}$$

In (1.1) $R(\varepsilon^{pl})$ is a hardening function, which is dependent on the equivalent plastic strain $\varepsilon^{pl}$. The yield function only depends in the deviator part of the stress tensor since $\boldsymbol{\sigma}_{eq}$ is given by:

$$\boldsymbol{\sigma}_{eq} = \sqrt{\frac{3}{2}\boldsymbol{I}_{dev} :: \langle\boldsymbol{\sigma}\rangle \otimes \langle\boldsymbol{\sigma}\rangle} = \sqrt{\frac{3}{2} dev(\boldsymbol{\sigma}) \otimes dev(\boldsymbol{\sigma})} \tag{1.2}$$

with:

$$\boldsymbol{I}_{dev} = \boldsymbol{I} - \boldsymbol{I}_{vol}, \quad \boldsymbol{I}_{vol} = \frac{1}{3}\mathbf{1} \otimes \mathbf{1} \tag{1.3}$$

In (1.2) $dev(\boldsymbol{\sigma})$ is the deviator of the stress tensor and the angular brackets indicate a homogenization relation for calculating the mean stress in a certain volume $\Omega$ of phase $p$:

$$\langle\boldsymbol{\sigma}\rangle^p = \frac{1}{\Omega^p}\int_{\Omega^p} \boldsymbol{\sigma}(x) \tag{1.4}$$

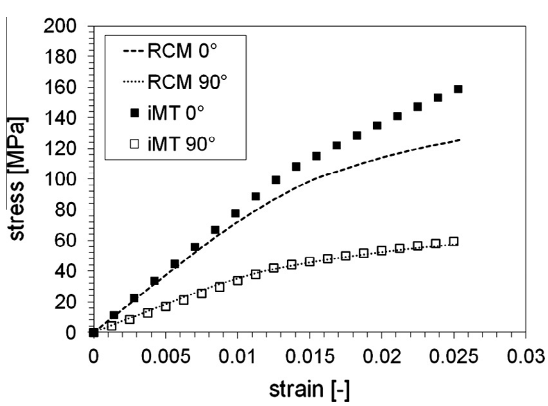


**Fig. 1.** Comparison of RCM and iMT simulations.

**Table 1**
Considered material data.

| | |
|---|---|
| Young's modulus matrix (MPa) | 2540 |
| Shear modulus matrix (MPa) | 940 |
| Poisson's ratio matrix (–) | 0.42 |
| Density matrix (g/cm$^3$) | 1.29 |
| Yield stress matrix (tension) (MPa) | 36 |
| Yield stress matrix (shear) (MPa) | 41 |
| Yield stress matrix (compression) (MPa) | 48 |
| Young's modulus fiber (MPa) | 72,000 |
| Poisson's ratio fiber | 0.22 |
| Density fiber (g/cm$^3$) | 2.54 |
| Aspect ratio (–) | 20 |
| Fiber weight fraction (%) | 20 |

Furthermore, in Eq. (1.3) $\boldsymbol{I}$ is the fourth order and $\mathbf{1}$ the second order identity tensor. The overestimation of the yield point in the iMT simulation is caused by only considering first moment estimates of the mean strain and stress fields (see e.g. Eq. (1.2), left part), whereas a realistic distribution is given in the RCM simulations. In the RCM FE model 50 fibers are modeled as ellipsoidal inclusions, which are embedded randomly in the matrix. The geometric fiber and mechanical fiber and matrix properties are in accordance to Table 1. Besides the periodic boundary conditions a displacement boundary condition is applied. To neglect a FE mesh or a boundary condition influence on the simulation results, the same FE model set up is used in all RCM and MT simulations. In Figs. 2 and 3 the stress distribution in the considered RCM model and the calculated stress by applying an iMT formulation are shown. The calculated von-Mises stress of the RCM model is classified into 50 classes of stress values and plotted against its fraction of occurrence. In Figs. 2 and 3 the von-Mises stress values with fraction 1, calculated with the iMT model, indicate what every FE element of the matrix phase has the same von-Mises stress. This is important to notice and one of the most critical shortcomings of analytical mean-field homogenization approaches. By calculating only one mean value the consideration of a realistic stress distribution, as shown by the RCM results, is not possible but required for a realistic plasticity or failure predictions. Recently, Doghri [1] proposed a second-moment incremental formulation for the Mori–Tanaka mean-field homogenization of composites (iMT-2) to improve this drawback. Therein, the second moment measure of the von-Mises yield stress $\sigma_{eq}^{2nd}$ is calculated by:

$$\sigma_{eq}^{2nd} = \sqrt{\frac{3}{2}\boldsymbol{I}_{dev} :: \langle\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}\rangle} \tag{1.5}$$

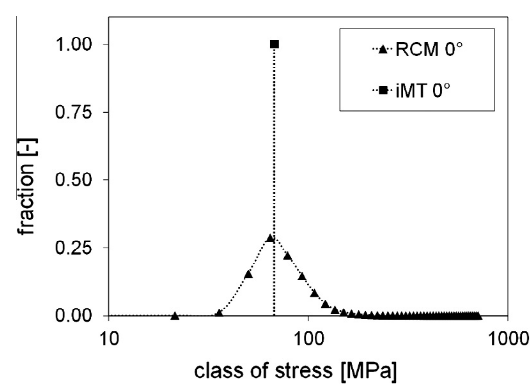


**Fig. 2.** Matrix stress distribution in a RCM and the mean matrix stress calculated by an iMT formulation by applying a load in fiber direction.

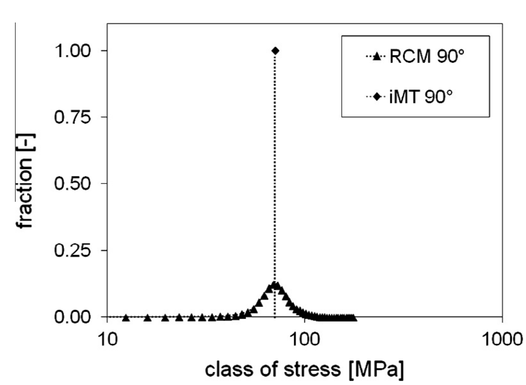


**Fig. 3.** Matrix stress distribution in a RCM and the mean matrix stress calculated by an iMT formulation by applying a load in transverse fiber direction.