Annals of Physics 337 (2013) 94–110

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Universality of entanglement creation in low-energy two-dimensional scattering[☆]

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HIGHLIGHTS

- The entanglement in low-energy scattering in two dimensions is universal. It is independent of the potential.
- We take the purity as the measure of entanglement.
- We give a rigorous computation of the leading order of the purity at low energy.
- The created entanglement depends strongly on the masses of the particles.
- It takes its minimum for equal masses and it increases strongly with the difference of the masses.

ARTICLE INFO

Article history:
Received 31 January 2013
Accepted 20 June 2013
Available online 27 June 2013

Keywords:
Entanglement
Low energy
Scattering
Purity
Gaussian state
Two dimension

ABSTRACT

We prove that the entanglement created in the low-energy scattering of two particles in two dimensions is given by a universal coefficient that is independent of the interaction potential. This is strikingly different from the three dimensional case, where it is proportional to the total scattering cross section. Before the collision the state is a product of two normalized Gaussians. We take the purity as the measure of the entanglement after the scattering. We give a rigorous computation, with error bound, of the leading order of the purity at low-energy. For a large class of potentials, that are not-necessarily spherically symmetric, we prove that the low-energy behavior of the purity, $\mathcal{P}$, is universal. It is given by $\mathcal{P} = 1 - \frac{1}{(\ln(\sigma/\hbar))^2}\,\mathcal{E}$, where σ is the variance of the Gaussians and the entanglement coefficient, $\mathcal{E}$, depends only on the masses of the particles and not on the interaction potential. There is a strong dependence of the entanglement in the difference of the masses. The

☆ Research partially supported by CONACYT under Project CB-2008-01-99100.

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0003-4916/$ – see front matter
http://dx.doi.org/10.1016/j.aop.2013.06.010

minimum is when the masses are equal, and it increases strongly with the difference of the masses.

1. Introduction

In this paper we consider the creation of entanglement in the low-energy scattering of two particles without spin in two dimensions. The interaction between the particles is given by a general potential that is not required to be spherically symmetric. Before the scattering the particles are in an incoming asymptotic state that is a product of two Gaussians. After the scattering the particles are in an outgoing asymptotic state that is not a product state. The problem that we solve is to compute the loss of purity, due to the entanglement with the other, that is produced by the collision.

In the configuration representation the Hilbert space of states for the two particles is $\mathcal{H} := L^2(\mathbb{R}^4)$. The dynamics of the particles is given by the Schrödinger equation,

$$i\hbar \frac{\partial}{\partial t} \varphi(\mathbf{x}_1, \mathbf{x}_2) = H\varphi(\mathbf{x}_1, \mathbf{x}_2). \tag{1.1}$$

The Hamiltonian is the following operator,

$$H = H_0 + V(\mathbf{x}_1 - \mathbf{x}_2), \tag{1.2}$$

where H_0 is the free Hamiltonian,

$$H_0 := -\frac{\hbar^2}{2m_1}\Delta_1 - \frac{\hbar^2}{2m_2}\Delta_2. \tag{1.3}$$

The operator Δ_j, is the Laplacian in the coordinates $\mathbf{x}_j, j = 1, 2$, of particle one and two. By $\hbar$ it is denoted Planck's constant. Furthermore, $m_j, j = 1, 2$, are, respectively, the mass of particle one and two, The interaction potential is a real-valued function, $V(\mathbf{x})$, defined for $\mathbf{x} \in \mathbb{R}^2$. We suppose that the interaction depends on the difference of the coordinates $\mathbf{x}_1 - \mathbf{x}_2$, but we do not require the spherical symmetry of the potential. We consider a general class of potentials that satisfy mild assumptions on its decay at infinity and on its regularity:

Assumption 1.1.

$$(1 + |\mathbf{x}|)^{\beta} V(\mathbf{x}) \in L^2(\mathbb{R}^2), \quad \text{for some } \beta > 11. \tag{1.4}$$

Under this assumption H is a self-adjoint operator.

We also suppose that at zero energy there is neither an eigenvalue nor a resonance (half-bound state), for the Hamiltonian of the relative motion $H_{\text{rel}} := -\frac{\hbar^2}{2m}\Delta_{\mathbf{x}} + V(\mathbf{x})$ where $\mathbf{x} \in \mathbb{R}^2$ is the relative distance and m is the reduced mass $m := m_1 m_2/(m_1 + m_2)$. A zero-energy resonance (half-bound state) is a bounded solution to $H_{\text{rel}}\varphi = 0$ that is not in $L^2(\mathbb{R}^2)$. See [1] for a precise definition. For generic potentials V there is neither a resonance nor an eigenvalue at zero for H_{rel}. That is to say, if we consider the potential λV with a coupling constant λ, zero can be a resonance and/or an eigenvalue for at most a finite or denumerable set of λ's without any finite accumulation point.

We study our problem in the center-of-mass frame. We consider an incoming asymptotic state that is a product of two normalized Gaussians, given in the momentum representation by,

$$\varphi_{\text{in},\mathbf{p}_0}(\mathbf{p}_1, \mathbf{p}_2) := \varphi_{\mathbf{p}_0}(\mathbf{p}_1)\, \varphi_{-\mathbf{p}_0}(\mathbf{p}_2), \tag{1.5}$$

with,

$$\varphi_{\mathbf{p}_0}(\mathbf{p}_1) := \frac{1}{(\sigma^2\pi)^{1/2}} e^{-(\mathbf{p}_1 - \mathbf{p}_0)^2/2\sigma^2}, \tag{1.6}$$

where $\mathbf{p}_i, i = 1, 2$ are, respectively, the momentum of particles one and two.