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Buckling and postbuckling of stiff lamellae in a compliant matrix

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ABSTRACT

Theoretical analysis and numerical simulations are performed to investigate the buckling and postbuckling behaviors of stiff lamellae embedded in a compliant matrix under uniaxial compression. First, the analytical solution is derived for the critical compressive strain and wrinkle wavelength of a stiff layer sandwiched between two different soft layers, in which the effects of interfacial shear stresses and matrix thicknesses have been taken into account. During postbuckling, the system may keep the sinusoidal buckling shape or bifurcate into period-doubling morphology. A phase diagram is established, which enables us to easily predict the morphological evolution from the geometric and material parameters of the system. Then the above analysis is extended to two or more parallel stiff lamellae within a compliant matrix. Different buckling modes are found in such multilayer systems, i.e., short-wave mode, long-wave mode, and hierarchical mode. This study not only sheds light on the stability and morphological evolution of lamella composites and structures but also helps understand the morphogenesis of some biological tissues.

1. Introduction

Sandwich composite structures consisting of stiff lamellae embedded in a compliant matrix are widely observed in geology [1–3], flexible or stretchable electronics [4], and biological tissues [5–7]. For example, some geological structures are composed of multiple layers with different thicknesses and mechanical properties. Due to crustal movements, wavy morphologies may form in the geological structures, which have been analyzed by using the soil–rock–soil model [1]. Some animal reflectors also have wavy multilayer structures. The crustacean of the crab *Ovalipes molleri* possesses a reflector composed of multiple solid layers within a very soft matrix, which has a function to broaden the reflectance band [6]. The keratinous sheaths of bovine horns have a multilayered structure with hierarchical wavy microstructures, which renders an enhanced toughening effect [7]. Besides, sandwich composite structures also find technological applications in thin-film metrology. They have some advantages over the traditional film-substrate systems, in which a stiff layer rests on a compliant substrate and interfacial delamination is apt to occur [8–10]. When a thin film or lamella is sandwiched between two soft layers, the bilateral confinement can efficiently prevent the occurrence of interfacial delamination. In engineering applications, on one hand, buckling represents a typical failure mode of sandwiched composite structures and should be avoided in service [4,11,12]. On the other hand, the instability of sandwich composite structures can be harnessed to control wave propagation, create band gaps, and filter undesirable frequencies, by varying the material, thickness, and spacing of the layers [13].

In the past decades, considerable attention has been paid to the buckling of traditional film-substrate structures [14–21]. When a stiff film lying on a compliant substrate is subjected to an in-plane compressive strain, the system may become unstable, rendering a sinusoidal surface morphology. The wrinkling wavelength λ obeys a scaling law of $\lambda \sim (E_f/E_s)^{1/3}$, where E_f and E_s are the elastic moduli of the thin film and substrate, respectively. This solution works only for a substrate of infinite thickness. For a thin substrate, the scaling law becomes $\lambda \sim (E_f/E_s)^{1/4}$, showing the significant role of the substrate thickness in the film buckling. With further increase in the compressive strain beyond the first buckling, the sinusoid wrinkling can be broken and progressively evolve into a period-doubling and even period-quadrupling pattern. To date, however, the buckling behavior of multilayered lamellae embedded in a compliant matrix remains elusive. Currie et al. [1] predicted the elastic buckling of a stiff beam embedded in a matrix by using the energy method. Parnes and Chiskis [4] developed a sandwich composite model to describe the elastic buckling of layer/fiber reinforced composites. Recently, Li et al. [22] considered the buckling of interfacial layers in stratified composites through

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theoretical analysis, experiments and finite element simulations. These previous studies assumed that the stiff beams or lamellae are embedded in an infinite matrix. In most practical structures and biological systems, however, the matrix is often finite in thickness.

In the present paper, therefore, we investigate, through theoretical analysis and numerical simulations, the buckling and morphological transition of sandwich structures consisting of stiff layers embedded in a soft matrix of finite thickness. This paper is organized as follows. In Section 2, a theoretical model is first presented to analyze the critical buckling of a lamella sandwiched between two compliant layers. The nonlinear postbuckling process is also examined to reveal the morphological transition in this system. In Section 3, we study the structures consisting of two or more parallel lamellae embedded in a soft matrix, which may buckle into different patterns. Finally, the main conclusions from this study are summarized in Section 4.

2. A lamella sandwiched between two soft matrix layers of different thicknesses

2.1. Theoretical model

First, consider a stiff lamella sandwiched between two compliant matrices of finite thicknesses, as shown in Fig. 1a. Refer to the Cartesian coordinate system (O–xy), where the origin O is located at the middle plane of the lamella, and the x and y axes are along and normal to the lamella, respectively. The thicknesses of the lamella, matrix-1 and matrix-2 are h, H_1 and H_2, respectively. In the critical buckling analysis, all materials are assumed to be linear elastic and isotropic so that the problem can be solved analytically. Denote the Young's moduli of the lamella, matrix-1 and matrix-2 as E, E_{m1} and E_{m2}, and their Poisson's ratios as v, v_{m1} and v_{m2}, respectively. The system is subjected to in-plane compressive strain ε along the x direction. Plane-strain conditions are assumed in the (x, y) plane. The lamella is flat when the nominal compressive strain ε is relatively small, and then it will buckle into a sinusoidal morphology when ε exceeds a critical value, ε_{cr}. Since the wrinkling wavelength of a lamella is generally much larger than its thickness, the von Karman nonlinear elastic plate theory is adopted for the lamella in this study. Linear perturbation analysis is first performed to predict the critical condition for the onset of buckling. The effect of interfacial shear stresses between the lamella and the matrix layers is taken into account. The equilibrium equations of the lamella are [8,18]

$$\frac{\bar{E}h^3}{12}\frac{d^4w}{dx^4} - \varepsilon\bar{E}h\frac{d^2w}{dx^2} = q_1 - q_2, \quad \bar{E}h\frac{d^2u}{dx^2} = -\tau_1 + \tau_2, \tag{1}$$

where u and w are the displacements in the x and y directions, $\bar{E} = E/(1 - v^2)$ is the plane-strain elastic modulus of the lamella, q_α and τ_α ($\alpha = 1, 2$) denote the normal and shear stresses at the interface between the lamella and matrix-α, respectively. We introduce sinusoid perturbations $w = w_m \cos(kx)$ and $u = u_m \sin(kx)$, where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, w_m and u_m are the amplitudes.

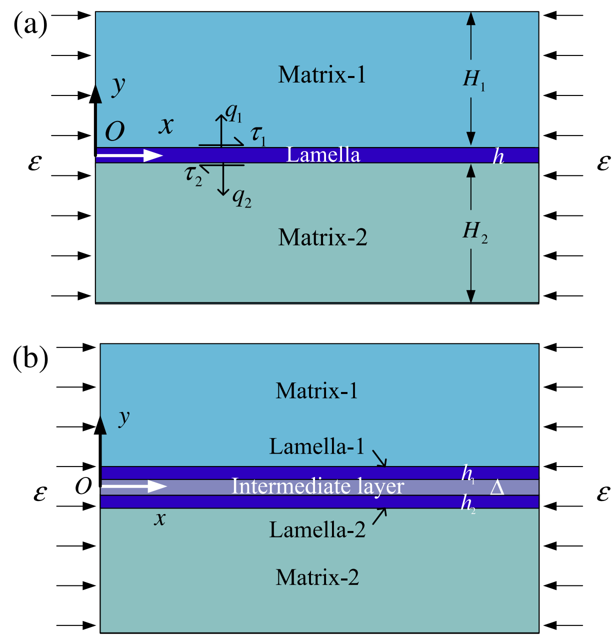

Fig. 1. (a) A stiff lamella sandwiched between two compliant layers (matrix-1 and 2), where q_α and τ_α ($\alpha = 1, 2$) denote the interfacial normal and shear stresses acting on the lamella, respectively. (b) A system consisting of two parallel lamellae in compliant matrices.

In the absence of body forces, the equilibrium condition of the matrix can be expressed as the Lamé–Navier equation:

$$(1 - 2v_{m\alpha})\nabla^2 u_i^{(\alpha)} + u_{j,ji}^{(\alpha)} = 0, \tag{2}$$

where $u_i^{(\alpha)}$ are the displacement components, and α takes the value of 1 and 2 for matrix-1 and 2, respectively.

For the considered multilayer system, global or Euler buckling may occur when the thicknesses of the matrices are small [23]. In the present paper, however, our attention is focused on the local buckling patterns and morphological evolution of one or more lamellae sandwiched between two compliant matrices. To exclude both rigid-body motions and global buckling, we specify the following boundary conditions:

$$\begin{cases} w^{(1)} = w_m\cos(kx),\ u^{(1)} = u_m\sin(kx) & \text{at } y = h/2, \\ \sigma_{yy} = 0,\ \sigma_{xy} = 0 & \text{at } y = h/2 + H_1, \end{cases} \tag{3}$$

$$\begin{cases} w^{(2)} = w_m\cos(kx),\ u^{(2)} = u_m\sin(kx) & \text{at } y = -h/2, \\ w^{(2)} = 0,\ \sigma_{xy} = 0 & \text{at } y = -h/2 - H_2. \end{cases} \tag{4}$$

Following the procedure presented in [24], Eq. (2) can be transformed into two ordinary differential equations of displacements $u^{(\alpha)}$ and $w^{(\alpha)}$ for each matrix. Then using the boundary conditions in Eqs. (3) and (4), the fields $u^{(\alpha)}$ and $w^{(\alpha)}$ can be solved. Further, the stresses at the lamella-matrix interfaces are obtained as $q_\alpha = q_{m\alpha}\cos(kx)$ and $\tau_\alpha = \tau_{m\alpha}\sin(kx)$, whereand $\bar{E}_{m\alpha} = E_{m\alpha}/(1 - v_{m\alpha}^2)$ is the plane-strain elastic moduli of matrix-α.

$$\begin{aligned} q_{m1} &= \frac{\bar{E}_{m1}(1 - v_{m1})k\left\{u_m[(1 - 2H_1^2k^2 - 2v_{m1}) + (-1 + 2v_{m1})\cosh(2H_1k)] + w_m[-4H_1k(-1 + v_{m1}) + 2(-1 + v_{m1})\sinh(2H_1k)]\right\}}{5 + 2H_1^2k^2 + 4v_{m1}(-3 + 2v_{m1}) + (3 - 4v_{m1})\cosh(2H_1k)}, \\ \tau_{m1} &= \frac{\bar{E}_{m1}(1 - v_{m1})k\left\{u_m[4H_1k(-1 + v_{m1}) + 2(-1 + v_{m1})\sinh(2H_1k)] + w_m\left[1 - 2(H_1^2k^2 + v_{m1}) + (-1 + 2v_{m1})\cosh(2H_1k)\right]\right\}}{5 + 2H_1^2k^2 + 4v_{m1}(-3 + 2v_{m1}) + (3 - 4v_{m1})\cosh(2H_1k)}, \end{aligned} \tag{5}$$