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Dynamics of nonautonomous rogue waves in Bose–Einstein condensate

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ABSTRACT

We study rogue waves of Bose–Einstein condensate (BEC) analytically in a time-dependent harmonic trap with a complex potential. Properties of the nonautonomous rogue waves are investigated analytically. It is reported that there are possibilities to 'catch' rogue waves through manipulating nonlinear interaction properly. The results provide many possibilities to manipulate rogue waves experimentally in a BEC system.

1. Introduction

Rogue waves (RW) are localized both in space and time and depict a unique event which seems to appear from nowhere and disappear without a trace. They are one of those fascinating destructive phenomena in nature that have not been fully explained so far [1–4], although there is a variety of approaches to deal with them [5–8]. Recently, RW have been observed experimentally in nonlinear optics [3] and a water wave tank [4]. It is shown that nonlinear theories can be used to explain the dramatic phenomena. Among nonlinear theories, the most fundamental is based on the nonlinear Schrödinger equation (NLS) [9]. The rational solution of NLS has been used to describe the RW phenomena [9,10].

Bose–Einstein condensate (BEC) represents a fluid, which is accurately described by the Gross–Pitaevskii (GP) equation in the mean-field approximation [11]. It is natural to expect that RW could exist in a BEC system [12]. For a BEC system, it is possible to manage the nonlinear interaction between atoms by Feshbach resonance technique, and there are many kinds of tunable atomic trapping potentials. These characters would provide us with a powerful tool to study properties or

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mechanism of RW. Therefore, it is meaningful to study RW solution of nonautonomous GP equations. Yan has solved a generalized nonautonomous NLS through the similarity transformation and direct ansatz [13]. Moreover, some scientists have investigated the formative mechanism of matter rogue wave in BECs with time-dependent interaction in an expulsive parabolic potential analytically and numerically [14]. The numerical results show that a small periodic perturbation with a smaller modulating frequency can induce the generation of the near-ideal rogue wave. Therefore, it is believed that the interesting phenomenon of RW in BECs can be observed experimentally [14]. Furthermore, we will discuss how to manipulate RW through managing related physical variables.

In this paper, we present a RW solution for BEC in a generalized time-dependent harmonic trap with time-dependent nonlinear interaction and a complex potential, through the Darboux transformation method. Furthermore, we observe dynamics and kinematics of rogue waves in detail for the first time, through calculating its shape and motion analytically. Based on the expressions which describe the evolution of RW's properties, and the compatibility condition, RW can be manipulated well. We find that RW can be long-lived through properly managing nonlinear parameter and gain term. It is shown that RWs still exist in an oscillating trap potential with oscillating nonlinear interaction parameter. These results are helpful to find ways to manage them experimentally in a BEC system.

2. The model and nonautonomous rogue waves solution

The management of nonlinearity and a tunable atomic trapping potential provide us with a powerful tool for manipulating a rogue wave. Therefore, it could be significant to study dynamics of RW of the following GP equation in BEC,

$$i\frac{\partial\psi(x,t)}{\partial t}+\frac{\partial^2\psi(x,t)}{\partial x^2}+2R(t)|\psi(x,t)|^2\psi(x,t)+M(t)x^2\psi(x,t)+i\frac{G(t)}{2}\psi(x,t)=0, \tag{1}$$

where $R(t)$ is nonlinearity management parameter which describes the variation of scattering length and can be controlled well by Feshbach resonance, $G(t)$ is appropriate gain ($G(t) < 0$) or loss ($G(t) > 0$) terms which can be phenomenologically incorporated to account for the interaction of the atomic cloud or thermal cloud. $M(t)x^2$ means a time-dependent harmonic trap. The similar equations have been solved exactly to present a soliton solution by many different methods in [15–17].

It is well known that there are always some integrable conditions to solve the nonlinear equation [15]. Here, we solve Eq. (1) under the integrable condition $R(t) = g\exp[\int G(t) - 4C(t)dt]$ and $M(t) = 4C(t)^2 + \frac{dC(t)}{dt}$. We can solve it to get the seed solution

$$\psi_0(x,t) = sA(t)B(t)\exp[i\theta(x,t)], \tag{2}$$

which can be seen as the background where RW appears. The parameter s determines the initial amplitude of the background, and in the seed solution

$$A(t) = \exp\left[\int -4C(t)dt\right],$$
$$B(t) = \exp\left[\int (-G(t)/2 + 2C(t))\,dt\right],$$
$$\theta(x,t) = C(t)x^2 + \int 2gs^2A(t)^2dt.$$

From the seed $\psi_0(x,t)$, we could present a nonautonomous RW solution in the generalized form through the Darboux transformation method [17] as

$$\psi(x,t) = \left[-1 + \frac{2 + i4\sqrt{g}sK(t)}{w(x,t)}\right]\psi_0(x,t), \tag{3}$$