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An analog model for quantum lightcone fluctuations in nonlinear optics

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ABSTRACT

We propose an analog model for quantum gravity effects using nonlinear dielectrics. Fluctuations of the spacetime lightcone are expected in quantum gravity, leading to variations in the flight times of pulses. This effect can also arise in a nonlinear material. We propose a model in which fluctuations of a background electric field, such as that produced by a squeezed photon state, can cause fluctuations in the effective lightcone for probe pulses. This leads to a variation in flight times analogous to that in quantum gravity. We make some numerical estimates which suggest that the effect might be large enough to be observable.

1. Introduction

A quantum theory of gravity is expected to predict quantum fluctuations of the spacetime geometry, and hence of the lightcone. This effect was recognized long ago by early workers on quantum gravity [1–3]. In particular, Pauli [1] once suggested that lightcone fluctuations might remove the ultraviolet divergences of quantum field theory by smearing the singular behavior of Green's functions. This hope has not yet been realized, but still remains a possibility, given that a complete quantum theory of gravity has not yet been found. Several proposals for the small scale structure of spacetime near the Planck scale have been made, including that of "spacetime foam" [4,5]. For a recent discussion of work on this topic, including observational bounds, see Ref. [6]. The possibility

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of an energy-dependent refractive index in one model was discussed in Ref. [7]. Even in the absence of a full theory, it is possible to discuss lightcone fluctuations in weak field quantum gravity. When linearized perturbations of a fixed background are quantized, it becomes possible to treat smearing of Green's functions and the associated lightcone fluctuations [8–10]. These are the *active* fluctuations, coming from the dynamical degrees of freedom of gravity itself. There are also *passive* fluctuations, which are driven by the fluctuations of the stress tensor of matter fields [11].

One expects quantum gravity effects to be very small, except in extreme conditions such as the early universe or near small black holes. For this reason, it is of interest to seek analog models in condensed matter systems, where similar effects might be exhibited but be much larger. The use of analog models for black hole evaporation has been a very active area of research, and is reviewed extensively in Ref. [12]. The effects of spatial randomness in a material have recently been treated as an analog model for lightcone fluctuations [13,14]. In the present paper, we wish to propose a different model based on nonlinear optics, where fluctuations of the effective dielectric function lead to a form of lightcone fluctuations.

There has long been interest in the parallels between light propagation in gravitational fields and in dielectric media. Gordon [15] showed that a gravitational field may be used to mimic the effects of a dielectric. Conversely, it is possible to define a dielectric medium which mimics the effect of a gravitational field [16]. In the latter case, the medium must have the property that the magnetic permeability is equal in magnitude to the dielectric permittivity in Gaussian units. This is not realized by any known material. Our interest is not in attempting to reproduce the detailed effects of gravity in a medium, but rather in the fact that a realistic material alters the speed of light in the material. When this speed can fluctuate, a form of lightcone fluctuations arises.

The outline of the paper is as follows: In Section 2, we review selected aspects of lightcone fluctuations in linearized quantum gravity. The crucial results from nonlinear optics which are needed will be discussed in Section 3. In Section 4 we introduce our analog model, and derive a key result for the variance in photon flight times in the presence of the lightcone fluctuations. Some numerical estimates are given in Section 5, where we discuss the possibility of a laboratory experiment to look for the lightcone fluctuation effect. Our results are summarized and discussed in Section 6. We will employ SI units, except in Section 2, where $\hbar = c = 1$ units are used.

2. Lightcone fluctuations in linearized quantum gravity

In this section, we review selected aspects of lightcone fluctuations in quantum gravity. First, let us recall the situation in classical general relativity theory, where the lightcone is a dynamical object determined by the spacetime geometry. A small perturbation of the geometry alters the lightcone. If we start with a background of Minkowski spacetime, and add small perturbations, the apparent speed of light as measured by the flat background metric can either increase or decrease. In the case of a weak field Schwarzschild metric, the change is a decrease, leading to the well-known Shapiro time delay [17]. The measurement of the time delay of radar signals passing near the sun is one of the better experimental tests of general relativity [18]. Similarly, a Schwarzschild metric with a negative mass would lead to a time advance, or an apparent increase in the speed of light compared to flat spacetime. Of course, the local speed of light is constant; it is the apparent speed measured over a finite distance which changes.

Now suppose that the linearized perturbations are quantized and are subject to quantum fluctuations. This will lead to fluctuations in the apparent speed of light and hence of the lightcone. Let the metric be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, and $h_{\mu\nu}$ is the quantized linear perturbation, which now becomes the graviton field operator. Let $\sigma(x, x')$ be the invariant interval function, defined as one-half of the squared geodesic distance between spacetime points x and x'. In flat spacetime, $\sigma = \sigma_0$, where

$$\sigma_0 = \frac{1}{2}(x - x')^2. \tag{2}$$