Annals of Physics 329 (2013) 133–145

Contents lists available at SciVerse ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

The coordinate coherent states approach revisited

Yan-Gang Miao *, Shao-Jun Zhang

School of Physics, Nankai University, Tianjin 300071, People's Republic of China

ARTICLE INFO

Article history:
Received 14 August 2012
Accepted 12 November 2012
Available online 23 November 2012

Keywords:
Noncommutative plane
Coordinate coherent state
Unruh effect
Hawking radiation

ABSTRACT

We revisit the coordinate coherent states approach through two different quantization procedures in the quantum field theory on the noncommutative Minkowski plane. The first procedure, which is based on the normal commutation relation between an annihilation and creation operators, deduces that a point mass can be described by a Gaussian function instead of the usual Dirac delta function. However, we argue this specific quantization by adopting the canonical one (based on the canonical commutation relation between a field and its conjugate momentum) and show that a point mass should still be described by the Dirac delta function, which implies that the concept of point particles is still valid when we deal with the noncommutativity by following the coordinate coherent states approach. In order to investigate the dependence on quantization procedures, we apply the two quantization procedures to the Unruh effect and Hawking radiation and find that they give rise to significantly different results. Under the first quantization procedure, the Unruh temperature and Unruh spectrum are not deformed by noncommutativity, but the Hawking temperature is deformed by noncommutativity while the radiation specturm is untack. However, under the second quantization procedure, the Unruh temperature and Hawking temperature are untack but the both spectra are modified by an effective greybody (deformed) factor.

* Corresponding author.
E-mail addresses: miaoyg@nankai.edu.cn, miao_yangang@yahoo.com.cn (Y.-G. Miao), sjzhang@mail.nankai.edu.cn (S.-J. Zhang).

0003-4916/$ – see front matter
doi:10.1016/j.aop.2012.11.003

1. Introduction

After the seminal work by Seiberg and Witten [1], the noncommutativity of spacetimes was revived and paid more and more attention to henceforth, for instance, it was believed [2–4] to be an indispensable ingredient for the quantization of gravity. In general there exist two common methods to deal with the noncommutative spacetime. The first one is the "star-product", which encodes the noncommutativity through replacing the ordinary product between functions by the Moyal–Weyl product [5–7]. This method has intensively been applied to the construction of field theories and gravity theories on noncommutative spacetimes. For reviews, see Refs. [8–11]. Within this framework one calculates only order by order in noncommutative parameters and then loses the nonlocality of noncommutative theories. Nevertheless, the second method, i.e. the "coordinate coherent states approach" [12–14] is quite different from the first one in the study of noncommutative quantum mechanics and quantum field theory. The models established in quantum field theory with such an approach are consistent with the Lorentz invariance (restricted in the Euclidean space), unitarity and UV-finiteness. The main idea of this approach is that the physical position of a point is represented by the mean value of its coordinate operators on coherent states. Associated with the quantization procedure based on the normal commutation relation between an annihilation and creation operators, the plane wavefunction which usually represents a "free point particle" gets deformed by a damping factor, and then the Feynman propagator of scalar fields acquires an extra damping factor. This indicates that a point mass can be described by a Gaussian function instead of the usual Dirac delta function, i.e. a point mass is smeared over the width $\sqrt{\theta}$, where θ is the noncommutative parameter. Black hole solutions with this Gaussian point source, known as "noncommutative inspired black holes", have been shown to possess some special properties [15–24].

However, the specific quantization procedure mentioned above is noncanonical, under which, instead of the commutator between a field and its conjugate momentum, the commutator between an annihilation and creation operators is imposed to take the normal form as a basic point. This gives rise to the result that the commutator between a field and its conjugate momentum takes a deformed form containing a damping factor. From the point of view of the canonical quantization of field theory, we argue in this paper the noncanonical quantization procedure by still following the coordinate coherent states approach but taking the canonical quantization procedure, and quite interestingly we have a significantly different result. That is, if we demand the commutator between the field and its conjugate momentum takes the canonical form as the starting point, the Feynman propagator will be the usual form as that in the commutative theory. This indicates that a point mass should still be described by the usual Dirac delta function. As a result, the concept of point particles is still valid if we deal with the spacetime noncommutativity by making use of the coordinate coherent states approach. We note that the commutator between an annihilation and creation operators now gets deformed in the canonical quantization procedure, which still leads to deformed modes as the representation of the spacetime noncommutativity.

In order to investigate the dependence on quantization procedures, we apply the two quantization procedures to the Unruh effect and Hawking radiation and find that they indeed lead to significantly different results. There has been some literature [25–31] concerning the issues based on the coordinate coherent states approach associated with the noncanonical quantization procedure. For instance, in Ref. [25] the Unruh effect on noncommutative spacetimes was studied by the introduction of the Unruh–DeWitt detector. It was obtained that the positive Wightman–Green function acquires an extra damping factor in momentum space and then the response rate is suppressed by the noncommutativity of spacetime. As a result, it was argued that the Unruh–DeWitt detector registers a temperature which is so greatly suppressed by the noncommutativity that it can be neglected. In addition, based on the quantum field theory in curved spacetime and the Bogoliubov transformation, it was claimed in Ref. [26] that the Hawking radiation spectrum is not deformed by the noncommutativity and the Hawking temperature cannot be neglected. We note that the different results caused by the specific treatments in Refs. [25,26] emerge from the noncommutativity of spacetime and such a difference disappears on the ordinary (commutative) spacetime. As for the reasons behind, see Ref. [26] for the details. We re-examine the issues by using the coordinate coherent states approach associated with both the noncanonical and canonical quantization procedures and