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The effect of inertia on the Dirac electron, the spin Hall current and the momentum space Berry curvature

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ABSTRACT

We have studied the spin dependent force and the associated momentum space Berry curvature in an accelerating system. The results are derived by taking into consideration the non-relativistic limit of a generally covariant Dirac equation with an electromagnetic field present, where the methodology of the Foldy–Wouthuysen transformation is applied to achieve the non-relativistic limit. Spin currents appear due to the combined action of the external electric field, the crystal field and the induced inertial electric field via the total effective spin–orbit interaction. In an accelerating frame, the crucial role of momentum space Berry curvature in the spin dynamics has also been addressed from the perspective of spin Hall conductivity. For time dependent acceleration, the expression for the spin polarization has been derived.

1. Introduction

In recent times, there has been growing interest in the field of *spintronics*. In this broad area of research, one studies the quantum transport properties of the electron spins and their technological applications [1–3]. As spin current is a non-conserved quantity, the control and generation of spin current is a challenging task. Since the theoretical prediction of the spin Hall effect (SHE) [4], the application of spintronics has seen considerable advancement. This effect is observed experimentally in semiconductors and metals [5]. The SHE is a form of anomalous Hall effect induced by spin. Here a beam of particles separates into up and down spin projections in the presence of a perpendicular electric field in analogy to the Hall effect case, where charges are separated in a beam passing through a perpendicular magnetic field. Besides this, the spin–orbit interaction opens up the possibility of manipulating electron (or hole) spin in non-magnetic materials by electrical means [6,7] and thus

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has attracted a lot of attention from theoreticians and experimentalists recently. The SHE that occurs due to the spin–orbit coupling (SOC) of electrons with impurities is known as extrinsic, whereas the intrinsic SHE is due to SOC in the band structure of a semiconductor without the presence of disorder; this has become an active area of research [8–14]. However, though the studies on the inertial effect of electrons have a long history [15–18], the contribution of the spin–orbit interaction (SOI) in accelerating frames has not been much addressed in the literature. Very recently, there has been an elegant attempt to extend the theory of spin current to the inertial frame [19]. A theory has been proposed describing the direct coupling of the mechanical rotation and spin current, and predicting the spin current generation arising from rotational motion [19,20].

On the other hand, the discovery of the Berry phase [21] has shed new light on areas relevant to the understanding of the origins of many physical phenomena, from an intriguing perspective. The Berry phase approach has been successfully adopted to explain various quantum mechanical/semiclassical aspects of different systems in condensed matter physics [22–27]. In particular, study of the momentum space Berry phase associated with the spin Hall effect [28–31] has become a topic of great recent interest. One of the avenues of investigation of the motion of spins through gauge theories entails the study of the gauge fields, naturally coupling to the spin. In the field of spintronics, the theories based on the Berry gauge theories provide a deeper understanding of the physical effects and their origin. In the semiclassical equations of motion, the Berry curvature corrections have broad impacts on transport properties [22]. In momentum space the Berry gauge connection due to spin–orbit coupling arises in spintronic, optical [32,33] and graphene systems [34,35].

In a recent paper [20], the spin dependent inertial force is studied in the absence of an external electromagnetic field. In this context, it would be interesting to study the generation of spin current in an accelerating frame with the presence of an electromagnetic field and explore the conditions for the appearance of momentum space Berry curvature. In this paper, we study the inertial spin Hall effect, which refers to the fact that we investigate the conditions for the spin dependent inertial forces and spin currents that appear in accelerating frames. In our formulation, we consider the fundamental Dirac Hamiltonian for a particle in a linearly accelerating frame [18]. As the dynamics of spin currents is closely related to the spin–orbit interaction (SOI), we consider the low energy limit of the Dirac equation.

The formalism that we adopt is explained in detail in Section 2. We have dealt with the idea [20] of interpreting the effect of linear acceleration on an electron as an induced effective electric field in this section. The spin–orbit interaction resulting from this induced electric field along with electric field due to the spin–orbit interaction generated from the external electromagnetic field produces the spin current in our system. In Section 2.4 we follow the physically intuitive approach of Chudnovsky [36], based on an extension of the Drude model which accounts for the spin and SOI, and derive the spin Hall current and conductivity. The connection of the SHE and the momentum space Berry curvature motivated us to explain the physical consequences of the momentum space Berry curvature in the inertial spin Hall effect, which is the content of Section 3. The paper ends with conclusions in Section 4.

2. The Dirac equation in a linearly accelerating frame

2.1. The Hamiltonian

Let us start by constructing the Dirac equation in a non-inertial frame, following the work of Hehl and Ni [18]. The essential idea in [18] is to introduce a system of orthonormal tetrads carried by the accelerating observer. This in turn induces a non-trivial metric and subsequently one rewrites the Dirac equation in the observer's local frame where normal derivatives are replaced by covariant derivatives derived from the induced metric. The Dirac Hamiltonian in an arbitrary non-inertial frame with linear acceleration and rotation for a charged particle having charge e is given by Hehl and Ni [18]:

$$H = \beta mc^2 + c\left(\vec{\alpha}\cdot\left(\vec{p}-\frac{e\vec{A}}{c}\right)\right) + \frac{1}{2c}\left[(\vec{a}\cdot\vec{r})\left(\left(\vec{p}-\frac{e\vec{A}}{c}\right)\cdot\vec{\alpha}\right) + \left(\left(\vec{p}-\frac{e\vec{A}}{c}\right)\cdot\vec{\alpha}\right)(\vec{a}\cdot\vec{r})\right] + \beta m(\vec{a}\cdot\vec{r}) + eV(\vec{r}) - \vec{\Omega}\cdot(\vec{L}+\vec{S}) \tag{1}$$