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Conforming discretizations of boundary element solutions to the electroencephalography forward problem

Discrétisations conformes des solutions aux éléments de frontière du problème direct en électroencéphalographie

Lyes Rahmouni [a,*], Simon B. Adrian [a,b], Kristof Cools [c], Francesco P. Andriulli [a,d,*]

[a] *IMT Atlantique, Technopole Brest-Iroise, 29238 Brest, France*
[b] *Technische Universität München, Arcisstr. 21, 80333 München, Germany*
[c] *The University of Nottingham, University Park, Nottingham, NG7 2RD, UK*
[d] *Politecnico di Torino, corso Duca degli Abruzzi 24, 10129 Torino, Italy*

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ABSTRACT

In this paper, we present a new discretization strategy for the boundary element formulation of the Electroencephalography (EEG) forward problem. Boundary integral formulations, classically solved with the Boundary Element Method (BEM), are widely used in high resolution EEG imaging because of their recognized advantages, in several real case scenarios, in terms of numerical stability and effectiveness when compared with other differential equation based techniques. Unfortunately, however, it is widely reported in literature that the accuracy of standard BEM schemes for the forward EEG problem is often limited, especially when the current source density is dipolar and its location approaches one of the brain boundary surfaces. This is a particularly limiting problem given that during an high-resolution EEG imaging procedure, several EEG forward problem solutions are required, for which the source currents are near or on top of a boundary surface.

This work will first present an analysis of standardly and classically discretized EEG forward problem operators, reporting on a theoretical issue of some of the formulations that have been used so far in the community. We report on the fact that several standardly used discretizations of these formulations are consistent only with an L^2-framework, requiring the expansion term to be a square integrable function (i.e., in a Petrov–Galerkin scheme with expansion and testing functions). Instead, those techniques are not consistent when a more appropriate mapping in terms of fractional-order Sobolev spaces is considered. Such a mapping allows the expansion function term to be a less regular function, thus sensibly reducing the need for mesh refinements and low-precisions handling strategies that are currently required. These more favorable mappings, however, require a different and conforming discretization, which must be suitably adapted to them. In order to appropriately fulfill this requirement, we adopt a mixed discretization based on dual boundary elements residing on a suitably defined dual mesh. We devote also a particular attention to implementation-oriented details of our new technique that will allow the rapid incorporation of our finding in one's own EEG forward solution technology. We conclude by showing how the resulting forward EEG problems show favorable properties with respect

* Corresponding author.
E-mail addresses: lyes.rahmouni@telecom-bretagne.eu (L. Rahmouni), simon.adrian@tum.de (S.B. Adrian), Kristof.Cools@nottingham.ac.uk (K. Cools), francesco.andriulli@polito.it (F.P. Andriulli).

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to previously proposed schemes, and we show their applicability to real-case modeling scenarios obtained from Magnetic Resonance Imaging (MRI) data.

R É S U M É

Dans ce papier, nous présentons une nouvelle stratégie de discrétisation pour la formulation aux éléments de frontière du problème direct de l'électroencéphalographie (EEG). Les méthodes aux éléments frontières (BEM) sont largement utilisées en imagerie EEG à haute résolution dans divers scénarios, pour leur stabilité numérique et leur efficacité reconnues par rapport à d'autres techniques basées sur des équations différentielles.
Malheureusement, il est également reconnu dans la littérature que leur précision diminue particulièrement lorsque la source de courant est dipolaire et se situe près de la surface du cerveau. Ce défaut constitue une importante limitation, étant donné qu'au cours d'une session d'imagerie EEG à haute résolution, plusieurs solutions du problème direct EEG sont requises, pour lesquelles les sources de courant sont proches ou sur la surface de cerveau.
Ce travail présente d'abord une analyse des opérateurs intervenant dans le problème direct et leur discrétisation. Nous montrons que plusieurs discrétisations couramment utilisées ne conviennent que dans un cadre L^2, nécessitant que le terme d'expansion soit une fonction de carré intégrable. Dès lors, ces techniques ne sont pas cohérentes avec les propriétés spectrales des opérateurs en termes d'espaces de Sobolev d'ordre fractionnaire.
Nous développons ensuite une nouvelle stratégie de discrétisation conforme aux espaces de Sobolev avec des fonctions d'expansion moins régulières, donnant lieu à une nouvelle formulation intégrale. Le solveur résultant présente des propriétés favorables par rapport aux méthodes existantes et réduit sensiblement le recours à un maillage adaptatif et autres stratégies actuellement requises pour améliorer la précision du calcul. Les résultats numériques présentés corroborent les développements théoriques et mettent en évidence l'impact positif de la nouvelle approche.

State-of-the-art high-resolution Electroencephalography (EEG) can righteously be considered a fully fledged imaging technique for the brain [1]. Its high temporal resolution, together with the compatibility and complementarity with other imaging strategies – Magnetoencephalography (MEG), Positron Emission Tomography (PET), and Magnetic Resonance Imaging (MRI) – [2–5], explains the steady interest that EEG is attracting in neuroimaging [6–8]. The peculiarity of high-resolution EEGs with respect to the traditional analyses based on grapho-elements, is the reconstruction of the volume brain sources based on scalp potential data [9,10]. This is the EEG inverse source problem, which is, as it is well known, ill-posed [11]. The solution to the EEG inverse source problem relies on multiple iterated solutions to the EEG forward problem where, known the configuration of brain sources, the electric potential is recovered at the scalp [12]. The accuracy in the solution to the EEG forward problem clearly impacts and limits the accuracy of the associated EEG inverse problem: a low accuracy of the solutions to the EEG forward problem translates in a low accuracy of the inverse problem solution [13]. This results in the pressing need to keep the accuracy of the EEG forward problem as high as possible.

Among the techniques to solve the EEG forward problem, Boundary Element Method (BEM) is a widely used one [14]. This numerical strategy is based on an integral formulation equivalent to the Poisson equation and, when compared with other numerical approaches like the Finite Element Method (FEM) or the Finite Difference Method (FDM) [15], BEM based solvers only discretize the surfaces enclosing the different brain regions and do not require the use of boundary conditions to terminate the solution domain. This results in interaction matrices of a smaller dimensionality [16] and explains the popularity of the BEM approach in the scientific community. Unfortunately, standard BEM methods are no panacea. It is widely reported, in fact, that the accuracy of standard BEM schemes for the forward EEG problem is often limited, especially when the current source density is dipolar and its location approaches one of the brain boundary surfaces [17,18]. This is a particularly limiting problem given that, during the solution to the EEG inverse source problem, several forward EEG problem solutions are required for which the the primary current density terms are near or on top of a boundary surface [19,20].

Three main strategies have been reported in the literature to limit the impact of accuracy losses: (i) the avoidance of brain source modeling near boundaries [21], (ii) the use of global or local mesh refinements that can better handle the singularity of the dipolar source term [22,23,20], and (iii) the introduction of a symmetric boundary element formulation [24,25]. All the above-mentioned techniques can sensibly improve source-related precision issues, but at the same time they present some undesirable drawbacks: (i) avoiding the positioning of dipolar sources near boundaries, on the one hand, represents a limitation on correct modeling [19] and, on the other hand, it increases the ill-posedness of the inverse-source