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Analysis on lump, lumpoff and rogue waves with predictability to the (2 + 1)-dimensional B-type Kadomtsev–Petviashvili equation

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ABSTRACT

In this work, we investigate the (2 + 1)-dimensional B-type Kadomtsev–Petviashvili (BKP) equation, which can be used to describe weakly dispersive waves propagating in the quasi media and fluid mechanics. We construct the more general lump solutions, localized in all directions in space, with more arbitrary autocephalous parameters. By considering a stripe soliton generated completely by lump solution, a lumpoff solution is presented. Its lump part is cut by soliton part before or after a specific time, with a specific divergence relationship. Furthermore, combining a pair of stripe solitons, we obtain the special rogue waves when lump solution is cut by double solitons. Our results show that the emerging time and place of the rogue waves can be caught through tracking the moving path of lump solution, and confirming when and where it happens a collision with the visible soliton. Finally, some graphic analysis are discussed to understand the propagation phenomena of these solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) have a widespread application in the filed of mathematical physics and engineering, such as fluid mechanics, plasma physics and optical fibers, etc. Finding exact solutions of NLEEs is one of the hot topics in this field. More and more researchers are interested in study this topic. Soliton solution, as a special exact solution, has an important position in the NLEEs. Over the development of the past decade, a number of methods have been presented to solve NLEEs, such as inverse scattering method [1], Hirota bilinear method [2], Darboux transformation (DT) [3] and Lie group method [4], etc. Recently, the researches of lump waves have attracted many attentions. As one kind of rogue waves with rational form, lump solution is localized in all directions in the space and appear in many physical phenomena, such as optical fibers, super-fluids and Bose–Einstein condensates [5–10]. Due to its important physical significance, the lump solution was first discovered in [11]. By employing Hirota bilinear method, a new direct method to get the lump solutions of KP equation was provided in [12]. It is a beautiful and wonderful method to seek lump solutions for nonlinear evolution equations. In terms of the direct method, the lump solutions for a lot of integrable equations have been derived, such as p-gKP equation, Yajima–Oikawa system [13–15], etc. Furthermore, the interaction solutions between lump waves and stripe solitons have been frequently reported in current research. There are many works to study the interaction solutions for the generalized KP equation [16], KP equation [17] and Ito equation [18], etc. These results indicate that the interaction solutions between the lump waves and a pair of stripe solitons could create the rogue waves at a special time.

In this work, we will consider the following (2+1)-dimensional B-type Kadomtsev–Petviashvili (BKP) equation [19]

$$u_t + u_{xxxxx} - 5\left(u_{xxy} + \int u_{yy} d_x\right) + 15\left(u_x u_{xx} + u u_{xxx} - u u_y - u_x \int u_y d_x\right) + 45u^2 u_x = 0, \tag{1.1}$$

where $u = u(x, y, t)$ is a analytic function with scaled spatial coordinates (x, y) and temporal coordinate t, the subscripts mean partial derivatives, and $\int$ is integration operator. The BKP equation, as a subclass of the KP hierarchy, can be used to describe weakly dispersive waves propagating in the quasi media and fluid mechanics. The multiple soliton solutions were established for Eq. (1.1) based on the simplified form of the Hirota method [20], and its periodic wave solutions with asymptotic behaviors were derived via using Riemann theta function in [21]. The soliton solutions, quasi-periodic wave solutions with their relations for the BKP equation

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were also reported in [22]. By using the extended homoclinic test method, rogue waves and homoclinic breather waves were provided in [23].

However, the main purpose of this work is to study the more general lump solutions, lumpoff solutions and predictable rogue wave solutions of the BKP equation (1.1) by using the symbolic calculation methods [24–50]. We first construct a general form of the lump solutions and analyze their moving path. Then, the lumpoff solutions, whose lump part can be cut by the soliton part during a specific time, are presented. The characteristics of lumpoff waves are that the soliton part is produced completely by the lump part. Moreover, when the lump waves are cut by a pair of stripe solitons produced by lump waves, a special rogue waves are generated. It is interesting that the special rogue waves can be forecasted by tracking where and when they emerge.

The outline of this work is as follows. In section 2, by constructing the general form of the lump solutions, we derive the general lump waves with the help of bilinear form for Eq. (1.1). In section 3, the lumpoff solutions of the equation are provided. In section 4, we obtain the special rogue solutions and indicate that they have predictability. Finally, some conclusions are given in the last section.

2. General lump solutions for BKP equation

First of all, we introduce a variable transformation

$$u = 2\,(\ln f)_{xx}\,, \tag{2.1}$$

where $f = f(x, y, t)$ is a function to be determined later. Substitution of (2.1) into (1.1) results in the following Hirota bilinear form for the BKP equation (1.1)

$$\left(D_x^6 - 5D_x^3 D_y - 5D_y^2 + D_x D_t\right) f \cdot f = 0, \tag{2.2}$$

with the D-operator defined by

$$D_x^l D_y^n D_t^m (f \cdot g) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^l \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^m \times f(x, y, t) \cdot g(x', y', t')\,|_{x=x', y=y', t=t'}. \tag{2.3}$$

The bilinear equation (2.2) has the following equivalent formula

$$2f_{6x}f - 12f_{5x}f_x + 30f_{4x}f_{xx} - 20f_{3x}^2 - 10f_{xxxy}f + 30f_{xxy}f_x - 30f_{xy}f_{xx} + 10f_y f_{3x} - 10f_{yy}f + 10f_y^2 + 2f_{xt}f - 2f_x f_t = 0. \tag{2.4}$$

If the solution f of above equation (2.4) is positive, the solution u defined by BKP equation (1.1) will be analytical based on the transformation in (2.1) (cf. [14]).

To get the single lump solution of BKP equation, we suppose a general quadratic function given by

$$f = x^T M x + f_0, \tag{2.5}$$

with

$$M = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix}, \qquad x^T = (x_0, x_1, x_2, x_3)\,, \tag{2.6}$$

where $f_0 \in R$ is a positive constant, $M \in R^{4\times 4}$ is a symmetric matrix, and $x^T \in R^4$ is a column vector and denotes the variables of function f. Letting $x_0 = 1$, $x_1 = x$, $x_2 = y$, $x_3 = t$, we can rewrite f as

$$f = \sum_{i,j=0}^{3} m_{ij}x_i x_j + f_0 = m_{11}x^2 + m_{22}y^2 + m_{33}t^2 + 2m_{12}xy + 2m_{13}xt + 2m_{23}yt + 2m_{01}x + 2m_{02}y + 2m_{03}t + m_{00} + f_0. \tag{2.7}$$

According to the transformation $u = 2(\ln f)_{xx}$, it is not hard to see that f must be positive. Therefore, $\sum_{i,j=0}^{3} m_{ij}x_i x_j$ should be non-negative. In order to make it, we redefine m_{ij} as follows

$$m_{ij} = \vec{P}_i \cdot \vec{P}_j = \sum_{k=1}^{n} P_{ik}P_{jk}, \tag{2.8}$$

where

$$\vec{P}_1 = \vec{a} = (a_1, a_2, \dots, a_n)\,, \qquad \vec{P}_2 = \vec{b} = (b_1, b_2, \dots, b_n)\,,$$
$$\vec{P}_3 = \vec{c} = (c_1, c_2, \dots, c_n)\,, \qquad \vec{P}_0 = \vec{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_n)\,, \tag{2.9}$$

are n dimension vectors and a_k, b_k, c_k, λ_k $(k = 1, 2, \dots, n)$ are real scalar parameters to be known later. Letting $\vec{\chi} = \sum_{i=0}^{3} x_i \vec{P}_i$, we have

$$\sum_{i,j=0}^{3} m_{ij}x_i x_j = \vec{\chi} \cdot \vec{\chi} = \sum_{k=1}^{n} \chi_k^2 \geq 0.$$

Thus f is always positive with m_{ij} defined by (2.8).

Substituting Eqs. (2.7)–(2.9) into equation (2.4) and collecting all the coefficients about x, y, t, we can obtain ten determining equations for m_{ij}. Solving these resulting equations based on (2.8), we can easily find the following constraints

$$c_k = \frac{-5\,(m_{22}a_k - 2m_{12}b_k)}{m_{11}},$$
$$f_0 = -m_{00} + \frac{m_{01}^2 m_{22} - 2m_{01}m_{02}m_{12} + m_{02}^2 m_{11}}{m_{11}m_{22} - m_{12}^2} - \frac{3m_{11}^2 m_{12}}{m_{11}m_{22} - m_{12}^2}. \tag{2.10}$$

Because the dimension n of the vectors $\vec{P}_i$ are undetermined, one can see that Eq. (2.7) with (2.8) should include a large number of parameters. Referring to the results presented in [51], we know that it is a more right choice for $n = 3$ to generate a more general lump solutions of Eq. (1.1). Therefore, taking $n = 3$, we rewrite constraint conditions as

$$c_1 = \frac{5\left[a_1\left(b_1^2 - b_2^2 - b_3^2\right) + 2b_1\left(a_2b_2 + a_3b_3\right)\right]}{a_1^2 + a_2^2 + a_3^2},$$
$$c_2 = \frac{5\left[a_2\left(b_2^2 - b_1^2 - b_3^2\right) + 2b_2\left(a_1b_1 + a_3b_3\right)\right]}{a_1^2 + a_2^2 + a_3^2},$$
$$c_3 = \frac{5\left[a_3\left(b_3^2 - b_1^2 - b_2^2\right) + 2b_3\left(a_1b_1 + a_2b_2\right)\right]}{a_1^2 + a_2^2 + a_3^2},$$
$$f_0 = -\frac{\left[\lambda_1(a_2b_3 - a_3b_2) + \lambda_2(a_3b_1 - a_1b_3) + \lambda_3(a_1b_2 - a_2b_1)\right]^2}{(a_1b_2 - a_2b_1)^2 + (a_2b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2} - \frac{3(a_1^2 + a_2^2 + a_3^2)^2\,(a_1b_1 + a_2b_2 + a_3b_3)}{(a_1b_2 - a_2b_1)^2 + (a_2b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2}, \tag{2.11}$$

where a_k, b_k, λ_k $(k = 1, 2, 3)$ are all arbitrary parameters.

Based on above analysis, the general lump solution u of BKP equation (1.1) can be expressed by