Physics Letters A 382 (2018) 2622–2625

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

A predictive framework for quantum gravity and black hole to white hole transition

Robert Oeckl

Centro de Ciencias Matemáticas, Universidad Nacional Autónoma de México, C.P. 58190, Morelia, Michoacán, Mexico

ARTICLE INFO

Article history:
Received 1 June 2018
Received in revised form 14 July 2018
Accepted 27 July 2018
Available online 1 August 2018
Communicated by M.G.A. Paris

Keywords:
Quantum foundations
Quantum gravity
Black hole
Quantum operation
Positive operator valued measure
Black hole bounce

ABSTRACT

The apparent incompatibility between quantum theory and general relativity has long hampered efforts to find a quantum theory of gravity. The recently proposed positive formalism for quantum theory purports to remove this incompatibility. We showcase the power of the positive formalism by applying it to the black hole to white hole transition scenario that has been proposed as a possible effect of quantum gravity. We show how the characteristic observable of this scenario, the bounce time, can be predicted within the positive formalism, while a traditional S-matrix approach fails at this task. Our result also involves a conceptually novel use of positive operator valued measures.

1. Introduction

Most approaches to quantum gravity rely substantially both on classical general relativity and on quantum theory. Quantum theory as it is usually understood relies on an a priori notion of time that is essential to the consistent interpretation of joint measurements. That is, the knowledge of the temporal order of measurements is a prerequisite for making predictions about their outcomes. On the other hand, in general relativity it is only the outcomes of measurements that reveal the spacetime structure and their temporal order. This incompatibility between core principles of general relativity and quantum theory in its usual form has posed a serious challenge for any attempt at bringing both theories together [1].

The most common approach to work around this problem has been to consider a situation where the strong gravity regime is confined to a compact spacetime region. Measurements take place only in an asymptotic region where gravity is weak and the metric is held fixed. This restriction appears to be physically well motivated and in close analogy to how measurements are defined in quantum field theory via the S-matrix. There, one assumes that particles can be approximated as free at very early and very late times in Minkowski spacetime, with interactions confined to intermediate times and treated perturbatively. Evidently, the perturbative treatment of the metric itself is more problematic than the perturbative treatment of other fields living on top of a fixed metric. It is well known that a straightforward quantum field theoretic treatment of perturbative general relativity fails due to non-renormalizability [2]. The example in this paper sheds further doubt on whether a perturbative approach in the spirit of the S-matrix to a gravitational theory can succeed even in principle.

It has been argued for some time [3] that the requirement for an absolute notion of time is not inherent to quantum theory, but an artifact of the *standard formulation* of quantum theory, which was conceived in the 1920s to resemble non-relativistic classical mechanics. A suitable, more fundamental framework for formulating quantum theory is now at hand in the form of the *positive formalism* [4–6]. This does not require an a priori notion of time and is fully compatible with the principles of general relativity, thus doing away with the apparent incompatibility. We demonstrate in this note how the positive formalism is capable of extracting predictions in quantum gravity where the conventional S-matrix approach fails. We focus on the example of a black hole to white hole transition and the associated bounce time.

2. A simple black hole bounce model

In general relativity, a black hole forms when sufficient mass density is reached. Spacetime acquires a singularity inside the black hole, signaling a break down of classical general relativity. It is widely believed that such singularities are an artifact of the

E-mail address: robert@matmor.unam.mx.

https://doi.org/10.1016/j.physleta.2018.07.041
0375-9601/

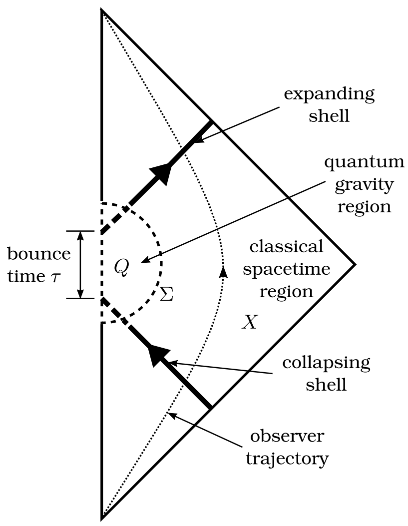

Fig. 1. Schematic Penrose diagram of black hole to white hole transition. A distant observer can infer the bounce time τ by observing the passage, first of the collapsing and then of the expanding shell.

purely classical treatment of gravity and will not be present in a quantum theory of gravity [7]. One proposed mechanism for avoiding singularities is that of a *bounce* [8]. That is, when in-falling matter starts to form a black hole, an effective repulsive force arises from quantum effects. This eventually leads to the formation of a white hole, which is a time reversed black hole, expulsing all matter to the surrounding spacetime. Note that this is distinct from the well established effect of Hawking radiation [9], which we neglect.

To model this process in the simplest possible way, we consider an in-falling spherically symmetric shell of matter with flat Minkowski spacetime in the interior. We further suppose that this shell is infinitesimally thin and contracts at the speed of light. Physically, there is only one relevant parameter that characterizes this contraction process, the mass-energy m of the shell. In classical general relativity a black hole of mass m would form and that would be the end of the story. We assume on the contrary that quantum effects cause the formation of an infinitesimally thin shell of matter of energy m that expands at the speed of light, leaving a flat Minkowski spacetime in the interior. While no metric satisfying Einstein's equations can describe this process in all of spacetime, it turns out that the initial black hole and final white hole metrics can be matched outside a "small" spacetime region that encloses the would-be singularity [10]. That is, the process conforms to general relativity everywhere, except in this spacetime region which we suppose to be governed by quantum gravity, see Fig. 1. What is more, the freedom in matching is described by a single parameter τ, called the *bounce time* [11]. A distant observer aware of its local spacetime geometry can measure the bounce time τ by registering the passages of the collapsing and the expanding shells. It corresponds to the time that would have elapsed between the end of the contraction of the collapsing shell to a point and the start of the expansion of the expanding shell from a point.

In order to formalize the problem, we divide spacetime into two regions, Q and X. Q is the strong gravity region, enclosed by the dashed line in Fig. 1. X is the weak gravity region outside of the dashed line that covers the remainder of spacetime. We denote by Σ the hypersurface that separates the two regions, indicated by the semicircular part of the dashed line. The classical physics in X is described by two parameters, the shell mass m and the bounce time τ. That is, the space of solutions L_X of the equations of motions in X can be written as $L_X = [0,\infty) \times [0,\infty)$. (The bounce time is taken to be bounded by 0 from below.) For simplicity we take the *phase space* or *space of initial data* L_Σ at the hypersurface Σ to be identical to L_X. (Generically one should expect L_X to restrict to a Lagrangian submanifold of L_Σ on the hypersurface [12].)

Suppose for the moment that rather than a quantum theory of gravity we considered a modified classical theory of gravity that would cause the bounce. This would yield for each shell mass m a bounce time $\tau_c(m)$. More formally, we would have a space L_Q of solutions of the classical equations of motions in Q. On the hypersurface Σ this would give rise to the subspace of L_Σ of those initial data that take the form $(m, \tau_c(m))$ for some m, which we shall also call L_Q. In this way L_Q is a 1-dimensional subspace of the 2-dimensional phase space L_Σ.

How can we predict the bounce time from a quantum theory of gravity? Suppose we follow the standard formulation of quantum theory and an S-matrix type approach. We should have a Hilbert space $\mathcal{H}$ of states of our system that describes its degrees of freedom well at least at early and at late times. At intermediate times interactions become important and in our case even metric spacetime itself ceases to exist as a classical entity. We then expect to describe this intermediate regime through an S-matrix $S : \mathcal{H} \to \mathcal{H}$. In the present case the initial and final states should describe the collapsing and the expanding shells respectively. However, viewed separately, neither the initial nor the final state carries any information about the bounce time. On the contrary, the states are the same for any bounce time. To obtain the bounce time we need an observer that continuously measures its surrounding spacetime metric, in particular using a clock. What is more, this is not a time measurement on a fixed metric background. Rather, the asymptotic metric is different for each bounce time and it is precisely this difference that encodes the bounce time.

3. Essentials of the positive formalism

The shortcomings of the S-matrix in a quantum gravity context provided an important motivation for the *general boundary formulation* as a new approach to the foundations of quantum theory [3]. The development of this approach [13], originally based on topological quantum field theory [14], has recently lead to the *positive formalism* [5,6].[1] This provides in particular a formulation of quantum theory that implements locality without requiring a metric spacetime background [4].

In short, the positive formalism is a framework for codifying physical theories by describing the possible *processes* occurring in them and provides a mechanism for predicting the outcomes of these processes. Processes include measurements, observations, interventions, but also "free evolution". In the *local* or *spacetime* version of the positive formalism of interest here, spacetime is cut up into *regions* so that a process is taken to occur in each region. The positive formalism then prescribes how these processes are *composed* and how outcomes for the resulting composite process are predicted. Crucially, at this level of description spacetime does not necessarily carry a fixed metric, but may have only a fixed topology.

In order to parametrize the possible interactions or "signals" between processes in adjacent spacetime regions, for each interfacing *hypersurface* Σ there is a set $\mathcal{B}^+_\Sigma$ of (proper) *boundary conditions*. Mathematically, this is the positive cone of a real partially ordered vector space $\mathcal{B}_\Sigma$ (of *generalized boundary conditions*). This comes from the fact that we are allowed to form new boundary conditions by probabilistically combining given ones with positive

[1] The formalism used in earlier papers on the general boundary formulation is now called the *amplitude formalism*. Its relation to the *positive formalism* is explained for bosonic theories in [4,6] and for fermionic theories in [4,15].