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Engineering solitons and breathers in a deformed ferromagnet: Effect of localised inhomogeneities

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A B S T R A C T

We investigate the soliton dynamics of the electromagnetic wave propagating in an inhomogeneous or deformed ferromagnet. The dynamics of magnetization and the propagation of electromagnetic waves are governed by the Landau–Lifshitz–Maxwell (LLM) equation, a certain coupling between the Landau–Lifshitz and Maxwell's equations. In the framework of multiscale analysis, we obtain the perturbed integral modified KdV (PIMKdV) equation. Since the dynamic is governed by the nonlinear integro-differential equation, we rely on numerical simulations to study the interaction of its mKdV solitons with various types of inhomogeneities. Apart from simple one soliton experiments with periodic or localised inhomogeneities, the numerical simulations revealed an interesting dynamical scenario where the collision of two solitons on a localised inhomogeneity create a bound state which then produces either two separated solitons or a mKdV breather.

1. Introduction

The problem of nonlinear excitations in ferromagnetic models has been under extensive investigations for many years due to its wide range of applications in real material systems [1–4]. For instance, ferromagnetic materials with different interactions have had a lot of importance in the context of data storage and allowed a faster coding of binary information. With these types of exchange interactions in the material, a localization phenomenon can be used to improve on these technological issues. Based on the availability of various nonlinear mechanisms in different physical fields, Ref. [5] has shown the importance of soliton dynamics for understanding physical phenomena and designing new experiments. Remarkable experiments in the past decade [6,7] used the propagation of electromagnetic (EM) waves in ferromagnetic mediums to demonstrate that magnetic field component of this EM wave could serve as a better option for the storage technology, using the fact that the magnetization can be manipulated to be aligned with a certain direction. This process is called magnetization reversal or switching and is possible only because the magnetization is localised as a soliton structure in both the space and time domain.

These qualitative features of the effect of EM waves on the magnetization dynamics can be understood theoretically by solving the associated nonlinear equations [8,9]. The soliton propagation in a ferromagnetic medium under the effect of EM wave was first studied by Nakata [8] using a multiscale approach on the celebrated Landau–Lifshitz equation coupled with the Maxwell equation. The magnetization dynamics in terms of solitary waves with long wavelengths is governed by the modified KdV equation. Extended studies were carried out by Leblond [10–15] who developed similar theories for several such applications. In these models, the dispersion relation has two possible phase velocities for the solitons, one case is studied in [8] and the other propagation mode was studied by Leblond in [11] and corresponds to KdV equation. In case of ferromagnetic slabs, a short solitary wave has been derived in [13] and the excitations are classified in polariton range. The background instability is suppressed by narrowing the slab and corresponds to the magnetization reversal.

When damping is included in the system, the nonlinear modulation and the solitons are governed by nonlinearity, dispersion and damping [14]. For small values of damping, the magnetization is governed by nonlinear Schrödinger (NLS) equation and the solitons cancel the effect due to damping. For large values of the damping, the wave decreases exponentially and no nonlinear modulation can occur and the dynamic is described by a perturbed NLS equation. In the three-dimensional case, the expected

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localised electromagnetic pulse was obtained in the focusing and defocusing cases from Davey–Stewartson (DS) equation admitting certain integrability conditions, as reported in [15]. The reduction to $(2+1)$-dimensional system is not integrable through the IST method for the hyperbolic–hyperbolic and elliptic–elliptic defocusing case. Corrections to the above model in the one-dimensional case have been obtained by taking into account Heisenberg exchanges, uniaxial anisotropies, the antiferromagnetic character of the medium and effects of damping due to the presence of free charges [16–18]. In these models, the propagation dynamics is described by the usual NLS family of equations. The effect of perturbation due to the presence of free charges in a conducting ferromagnetic medium [17] shows structural perturbations on the soliton, such as for example in [14].

In the studies we just mentioned, the focus was mainly on the magnetization of the medium and not on the wave evolution. However, waves should not be excluded when an exchange coupling is introduced in the ferromagnetic medium. In this setting, nonlinear excitations in classical models with significant exchange couplings have been studied widely in the past for the existence of solitons [19–30]. These investigations are devoted to the excitations and soliton solutions that are affected by the EM waves. Hence, it is necessary to exploit the excitations including the higher order exchange interactions.

Recently, the present author investigated the helimagnetic system in analogy to cholesteric liquid crystal and showed that EM waves are modulated in the form of solitons described by generalized derivative NLS equation [31]. In this letter, the inhomogeneous or site-dependent exchange model is considered for the study of soliton dynamics when coupled with the EM wave propagation.

1.1. Structure of the paper

In section 2 of this paper, we present the model and discuss the inhomogeneity of magnetic materials. In section 3 the dynamical system is reduced to the perturbed integral modified KdV equation using the multiscale analysis. Section 4 is devoted to solving the PIMKdV equation using numerical simulations and several effects of inhomogeneities are discussed, in particular periodic, and localised inhomogeneities, with the new process of emergence of breathers from soliton collisions on a wall. The summary of the investigation is presented in section 5.

2. Inhomogeneous magnetic system

2.1. The Maxwell–Landau equation

We use a coupling between the classical Landau–Lifshitz model for the magnetization density function $\mathbf{M}(x,t) = (M^x, M^y, M^z)$ and Maxwell's equations for the propagation of EM waves in ferromagnetic chains, see [32]. This results in the celebrated Maxwell–Landau (ML) model taking into account the Heisenberg exchange couplings and inhomogeneities while neglecting anisotropies, given by

$$\frac{\partial \mathbf{M}}{\partial t} = \mathbf{M} \wedge \left[J(f\mathbf{M}_{xx} + f_x \mathbf{M}_x) + \gamma \mathbf{H} \right], \tag{1}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\mathbf{H}+\mathbf{M}) = -\nabla(\nabla\cdot\mathbf{H}) + \nabla^2 \mathbf{H}, \tag{2}$$

where the constant J is the exchange integral, γ is the gyromagnetic ratio and the function f captures the inhomogeneities. In general f is a function of both space and time, such as $f = \mu_1(t)x + \nu_1(t)$ but the physical interpretation as a model for a ferromagnet is less clear for time-dependent functions. f is therefore taken as a time-independent function [33]. Finally, $c^2 = 1/\sqrt{\mu_0\varepsilon_0}$, where μ_0 is the magnetic permeability and ε_0 is the dielectric constant of the medium. The external magnetic field is coupled via the magnetic field component $\mathbf{H}$ of the EM wave. Since this model is inhomogeneous we only focus here on the one-dimensional study of the above equations (1) and (2) and leave the study of this higher dimensional equation for future works.

2.2. On inhomogeneities

Inhomogeneities otherwise known as deformations in a system may either be due to external fields or to the presence of defects, voids and gaps in the material. In the first case, inhomogeneities arise when a ferromagnetic medium lying in the $(x-y)$ plane is magnetized either in the longitudinal axis say x direction or transverse axis say y direction. When saturated along the longitudinal axis, a medium exhibits a homogeneous effective field $\mathbf{H}_{\text{int}}$ but when magnetized in the transverse axis, the induced effective field $\mathbf{H}_{\text{int}}$ is inhomogeneous, see [34]. These types of deformations are called field deformations. In the other case, a site-dependent function f is introduced into the Hamiltonian to model the lattice defects such that the corresponding exchange integral is bond dependent, see for example [35]. These lattice defects introduce a lattice distortion thereby leading the material to a deformed one. Such a dependence can occur if (a) the distance between neighbouring atoms varies along the chain, hence altering the overlap of electronic wavefunctions (assumed to be identical at all sites), or (b), if the wavefunction itself varies from site to site, even for equally spaced atoms. As examples of the case (a), we can mention charge transfer complexes TCNQ [36] where the inhomogeneity function f acting as a coefficient of exchange interactions is a set of random variables. Another example arises in organometallic insulators TTF-bisdithiolenes [37] for which f randomly alternates between two values along the chain. A chain system is natural for modelling inhomogeneities due to defects, although it may still be applicable in the case of weakly disordered systems with peaked wavefunctions such that a small change in the lattice constant causes a relatively large change in the atom overlap. These inhomogeneities can be modelled in the effective Hamiltonian for a one-dimensional magnetic insulator placed in a weak, static, inhomogeneous electric field or by the introduction of imperfections (impurities or organic complexes) in the vicinity of a bond to alter the electronic wavefunctions without causing appreciable lattice distortions. By gradually changing the concentration of impurities along the chain, it is possible to engineer a controlled inhomogeneity function f.

3. Dynamics of Maxwell–Landau model

3.1. Scaling

We will now use the approach of multiscale analysis to reduce the coupled vector equations (1) and (2) to a nonlinear equation for a scalar field $u(x,t)$. The multiscale analysis or the reductive perturbation method is a generalized asymptotic analysis method for solving the Maxwell–Landau model by reducing it to soliton equations and possibly integrable equations [16–18,38]. We first introduce the following slow space and time variables, depending on a small parameter ε and two exponents m and n,

$$\zeta = \varepsilon^m(x - Vt), \qquad \tau = \varepsilon^n t. \tag{3}$$

The slow space variable ζ describes the shape of the pulse propagating at speed V and the time variable τ is the time evolution of this pulse during the propagation. The parameter ε measures the weakness of the perturbation effect.