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Coherence resonance in an excitable potential well

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ABSTRACT

The excitable behaviour is considered as motion of a particle in a potential field in the presence of dissipation. The dynamics of the oscillator proposed in the present paper corresponds to the excitable behaviour in a potential well under condition of positive dissipation. Type-II excitability of the offered system results from intrinsic peculiarities of the potential well, whose shape depends on a system state. Concept of an excitable potential well is introduced. The effect of coherence resonance and self-oscillation excitation in a state-dependent potential well under condition of positive dissipation are explored in numerical experiments.

1. Introduction

The phenomenon of coherence resonance was originally discovered for excitable systems [1–6] and then for non-excitable ones [7–10]. This effect implies that noise-induced oscillations become more regular for an optimal value of the noise intensity. Coherence resonance is a frequent occurrence in neurodynamics [2,11,4,12] as well as in microwave [13] and semiconductor [14,15] electronics, optics [16–20], thermoacoustics [21], plasma physics [22], and chemistry [23–25].

The noise-induced dynamics of excitable systems in the regime of coherence resonance is similar to the self-oscillatory behaviour. The similarity is complemented by the fact that noise-induced oscillations corresponding to the coherence resonance can be synchronized mutually or by external forcing [26–28]. Moreover, the synchronization of the noise-induced oscillations occurs in a similar way as for a deterministic quasiperiodic system [29].

Similarity of nature of self-oscillation excitation and excitability is also seen in the context of interpretation of the dynamics as motion of a particle in a potential field in the presence of dissipation. In such a case, a mathematical model of the system under study is presented in the following oscillatory form:

$$\frac{d^2y}{dt^2}+\gamma\frac{dy}{dt}+\frac{dU}{dy}=0, \tag{1}$$

where the factor γ characterizes the dissipation, U is a function denoting the potential field. Mathematically, a paradigmatic model for type-II excitability is the FitzHugh-Nagumo system [30,31]:

$$\begin{cases} \varepsilon\dfrac{dx}{dt}=x-\dfrac{x^3}{3}-y, \\ \dfrac{dy}{dt}=x+a+\xi(t), \end{cases} \tag{2}$$

where the parameter ε sets separation of slow and fast motions, the parameter a determines the oscillatory dynamics, $\xi(t)$ is a source of noise. In the oscillatory form (1) the model (2) becomes:

$$\varepsilon\frac{d^2x}{dt^2}+\left(x^2-1\right)\frac{dx}{dt}+x+a=-\xi(t), \tag{3}$$

and describes stochastic motion of a point mass in the potential $U(x)=\frac{x^2}{2\varepsilon}+\frac{a}{\varepsilon}x$ in the presence of dissipation defined by the function $\gamma=\gamma(x)=\frac{x^2-1}{\varepsilon}$. In case $|a|<1$ the dissipation function $\gamma(x)$ possesses negative values in the vicinity of a local minimum of a potential well. It denotes energy pumping, which leads to instability of an equilibrium point, and self-sustained oscillation excitation occurs [Fig. 1 (a)]. Energy balance between dissipation and pumping during the period of the self-oscillations is organized after short transient time. The same self-oscillation excitation principle works in the Van der Pol [32] and Rayleigh [33] systems and in other examples of self-oscillators realizing the Andronov–Hopf bifurcation as well as in excitable systems with type-I excitability, where transition to the self-oscillatory regime is associated with the saddle-node bifurcation (see for example the two-dimensional

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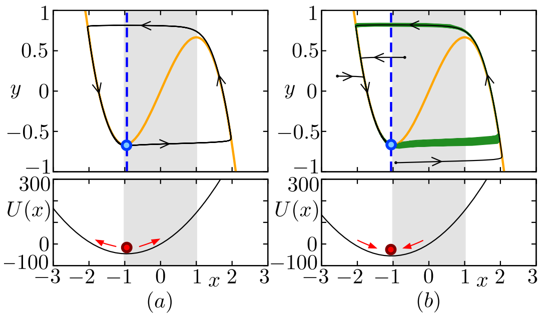

Fig. 1. Self-oscillatory (a) and excitable (b) regimes of the system (2) in numerical simulation: phase space structure (upper panels) and corresponding potential function $U(x)$ (lower panels). The equilibrium point is shown by the blue circle; the blue dashed line indicates the nullcline $\dot{y}=0$; the orange solid line shows the nullcline $\dot{x}=0$. Phase trajectories of the deterministic system (in case $\xi(t)\equiv 0$) are shown by black arrowed lines. Intrawell oscillations of a particle (the red circle on the lower panels) in the absence of noise are schematically shown by the red arrowed lines. The area corresponding to negative values of the dissipation function $\gamma(x)$ is coloured in grey. The green trajectory on the panel (b) corresponds to the system driven by white Gaussian noise ($\xi(t)=\sqrt{2D}n(t)$, $<n(t)>=0$, $<n(t)n(t+\tau)>=\delta(\tau)$, D is the noise intensity, $D=10^{-3}$). Parameters are: $\varepsilon=0.01$, $a=0.95$ (self-oscillatory regime), $a=1.05$ (excitable regime). (For interpretation of the colours in the figure(s), the reader is referred to the web version of this article.)

modification of the Hindmarsh-Rose neuron model [34]). If changing of the parameter a in the system (2) shifts the equilibrium point out of the negative dissipation area [grey areas in Fig. 1], then the steady state becomes stable and the system does not exhibit the self-oscillatory behaviour [Fig. 1 (b)]. However, in the presence of noise the force $\xi(t)$ randomly kicks the phase point out of the vicinity of the stable equilibria towards the region of the negative friction $\gamma(x)$. Phase point drift is amplified in the areas of negative dissipation. If energy pumping is sufficient, the phase point trajectory forms a loop [the green trajectory in Fig. 1 (b)]. Thus, both self-oscillation excitation and excitability are associated with the presence of negative dissipation[1] in the context of motion of a particle in a potential field.

The presented above interpretation of the self-oscillatory dynamics in terms of motion of a particle in a potential field involves the presence of a potential with a fixed profile and state-dependent dissipation, which possesses negative and positive values and is responsible for the existence of the self-sustained oscillations. Another configuration of self-oscillatory motion in the potential field is proposed in the paper [35]. It implies positive dissipation factor and a state-dependent potential, which is responsible for self-oscillation excitation. In that case both steady state instability and amplitude limitation are dictated by the potential function. This configuration corresponds to mutual interaction of the point mass particle and the potential field. The potential determines the particle's dynamics, but it changes depending on the velocity of the mass point. The publication [35] is focused on the dynamics of deterministic systems and does not involve study of the stochastic behaviour. The next step, which allows to expand a manifold of possible phenomena in the state-dependent potential well under condition of positive dissipation, is consideration of noise-induced effects. In particular, the effect of coherence resonance can be explored.

In the current paper we propose an excitable oscillator being similar to the model offered in the paper [35]. The dynamics of the system under study can be interpreted as motion of a particle in a state-dependent potential well in the presence of positive dissipation. The explored system exhibits the effect of coherence resonance associated with the type-II excitability. In contrast to the system (2) and other systems the excitable dynamics of the system under study is fully defined by the nonlinearity of a potential function and does not result from properties of a dissipation function. Therefore, the observed behaviour can be interpreted as motion of a particle in an excitable potential well. In the present paper we demonstrate that the effect of coherence resonance can be achieved in the state-dependent potential well under condition of positive dissipation. By this way we complement the results of the paper [35].

The present paper is organized as follows. In the section 2 the studied system is described in details. Then the effect of coherence resonance is shown in the noise-driven system (section 3). Conclusions are presented in the section 4.

2. Model and methods

Fig. 2 (a) shows a schematic circuit diagram, which mathematical model is derived below. It is a series-oscillatory LCR-circuit driven by a nonlinear feedback. The nonlinear feedback is realized by the nonlinear converter F. The converter F has two inputs V (the voltage across the capacitor C) and V_R (the voltage across the resistor R) with zero input current. In that case the same current i passes through the inductor L, the resistor R, the capacitor C and then $V_R=iR$. The converter F has the output voltage as being: $V_{out}=\frac{V_g}{(V-kV_R)^2+1}-V_a+\xi(t)=\frac{V_g}{(V-mi)^2+1}-V_a+\xi(t)$, where $m=kR$, $V_a=\text{const}$, $V_g=\text{const}$, $\xi(t)$ is the stochastic term determined by a broadband noise source included into the converter F. By using the Kirchhoff's voltage law the following differential equations for the voltage V across the capacitor C can be derived:

$$CL\frac{d^2V}{dt'^2}+RC\frac{dV}{dt'}+V=\frac{V_g}{\left(V-m\frac{dV}{dt'}\right)^2+1}-V_a+\xi(t). \tag{4}$$

In the dimensionless form with variable $x=V/v_0$ and time $t=t'/m$, where $v_0=1$ V, Eq. (4) can be re-written as,

$$\varepsilon\ddot{x}+\gamma\dot{x}+x+a-\frac{g}{(x-\dot{x})^2+1}=\sqrt{2D}n(t), \tag{5}$$

where $\dot{x}=\frac{dx}{dt}$, $\ddot{x}=\frac{d^2x}{dt^2}$, parameters are $\varepsilon=\frac{CL}{m^2}$, $\gamma=\frac{RC}{m}$, $a=V_a/v_0$, $g=V_g/v_0^2$, $n(t)$ is normalized Gaussian white noise ($<n(t)>=0$, $<n(t)n(t+\tau)>=\delta(\tau)$), and D is the noise intensity. Numerical modelling of the system (5) was carried out by integration using the Heun method [36] with the time step $\Delta t=0.001$.

One can imagine mechanical realization of the system (5). One of the simplest mechanical oscillators is a spring-based pendulum [Fig. 2 (b)]. In a linear pendulum case a spring-suspended solid is affected by the force of gravity, $\vec{F}_1=m\vec{g}$, and the elastic force, $\vec{F}_2$, being proportional to displacement $\vec{x}$ from equilibrium: $\vec{F}_2=-k\vec{x}$. Taking into account the influence of air resistance assumed to be proportional to the velocity of motion $\vec{F}_3=-\gamma\vec{v}$, one can derive an equation of motion in the vectorial form (Newtown's second law): $m\vec{a}=\vec{F}_1+\vec{F}_2+\vec{F}_3$. Then the scalar form of the equation is: $m\ddot{x}+\gamma\dot{x}+kx=0$. Generally, elastic properties of objects can be more complex (see for example a model of hair bundles with negative stiffness [37,38]). Moreover, the spring can be changed to a complex device [Fig. 2 (c)] including elastic medium (the medium S in Fig. 2 (c)) and a source of energy (is marked by

[1] It is important to note that the definition "positive" or "negative dissipation" determined by the sign of the term γ in the oscillatory forms (1) and (3) is correct only in terms of description of the dynamics as motion of a particle in a potential field. Generally, dissipativity of dynamical systems is determined by the divergence of the phase space flow.

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