Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Effects of rotation on Landau states of electrons on a spherical shell

Jonas R.F. Lima [a,*], Antônio de Pádua Santos [a], Márcio M. Cunha [a,b], F. Moraes [a,c]

[a] Departamento de Física, Universidade Federal Rural de Pernambuco, 52171-900, Recife, PE, Brazil
[b] Departamento de Matemática and Departamento de Física, CCEN, Universidade Federal de Pernambuco, 50670-901, Recife, PE, Brazil
[c] Departamento de Física, CCEN, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-900, João Pessoa, PB, Brazil

ARTICLE INFO

Article history:
Received 25 March 2018
Received in revised form 27 June 2018
Accepted 17 July 2018
Available online xxxx
Communicated by M.G.A. Paris

Keywords:
Inertial effects
Curved surface
Landau levels

ABSTRACT

Based on a previously observed analogy between electromagnetic and non-inertial effects, we investigate the competition between magnetic field and rotation in the quantum motion of an electron constrained to the surface of a sphere. We solve numerically the Schrödinger equation of the problem for the energy eigenvalues and the eigenfunctions and compare the effects of the magnetic field and rotation. We obtain that, for a weak magnetic field, an electron can not distinguish between magnetic field and rotation, since they lead to equivalent behavior. But this is no longer true for strong magnetic fields. However, surprisingly, even though the rotation and magnetic fields play different roles in the electronic properties of the system, when together, each influence of the magnetic field on the energy levels can be separately balanced by rotation. We also show that no matter the intensity of the magnetic field, it is always possible to destroy the Landau levels in the sphere by rotation.

1. Introduction

The influence of electromagnetic fields on quantum particles is a well known subject, and has been investigated since the birth of modern quantum mechanics through the inclusion of a scalar and a vector potential in the Schrödinger equation, in the context of non-relativistic quantum mechanics [1,2], or in the Dirac equation, in the context of relativistic quantum mechanics [3,4]. In a seminal work, Aharonov and Bohm proposed a new significance to the vector potential [5], opening several possibilities to the study of the influence of electromagnetic potentials on the quantum regime, even if the fields are not present. This is the Aharonov–Bohm effect, that can occur both in scattering and in bound states as well [6,7]. It was first obtained by Landau, for the case of Schrödinger equation, that electrons in two dimensions under a perpendicular magnetic field have a discrete energy spectrum, the so-called Landau levels [8]. These discrete levels are associated with a quantization of the cyclotron orbits expected in the classical regime. Since then, the investigation of the Landau levels in different systems has attracted great deal of attention. This interest, for instance, is due to the fact that some effects induced by magnetic fields, such as magnetic quantum oscillations [9–11] and the quantum Hall effect [12], can be better understood in terms of the Landau levels. Landau quantization in curved surfaces is also a topic of great research interest, as can be seen for example in Refs. [13–15]. In particular, the investigation of the Landau levels on a spherical surface was already addressed in different systems [16–19].

Various works have suggested an analogy between rotation and electromagnetic fields, since the inertial forces enter the Schrödinger Hamiltonian as a vector and a scalar potential, as do the electromagnetic potentials. For instance, Aharonov and Carmi proposed an inertial effect analogous to the Aharonov–Bohm (AB) effect [20,21]. They showed that the inertial vector potential can create a phase in the quantum states of charged particles, similar to the AB case. This effect was studied for valence electrons in rotating C60 molecules [22]. It is known that such molecules can rotate with frequencies around 10^{11} Hz [23]. Also, a Hall-like effect induced by rotation was investigated in Ref. [24]. The effects of rotation in Bose–Einstein Condensates (BEC) are analyzed in Refs. [25,26]. In this context, it is shown that the rotation can behaves like an effective magnetic field, providing conditions to phase transitions, for example. An experimental realization of a rotating BEC is described in Ref. [27]. In [28], the stability of stationary states in rotating nanostructures was studied, and the conditions for experimental verifications are discussed. In Ref. [29], rotational effects in the context of quantum interference were reported. The influence of rotation was also analyzed in the electronic structure of carbon nanomaterials, such as fullerene [30,31] and carbon nanotubes [32], where it was obtained that the spin-rotation coupling works like the Zeeman interaction, breaking the spin degeneracy, which suggest the possibility of inducing the spin Hall effect in graphene [33] by rotation. The experimental realiza-

* Corresponding author.
E-mail address: jonas.lima@ufrpe.br (J.R.F. Lima).

https://doi.org/10.1016/j.physleta.2018.07.029
0375-9601/

tion of rotating carbon nanotubes obtained from circularly polarized light is addressed in Refs. [34,35].

In this paper, motivated by this analogy, we investigate the interplay of magnetic field and rotation effects on the quantum motion of an electron constrained to the surface of a conducting sphere, a spherical shell quantum dot. We write out the Schrödinger equation for the problem and solve it for the energy eigenvalues and eigenfunctions. In order to understand the influence of the magnetic field and rotation individually, we first consider each one separately and subsequently solve the general case. The differences and similarities between rotation and magnetic field are then discussed. We verify that, for a weak magnetic field, the rotation is equivalent to the magnetic field in the electrons point of view. However, for a strong magnetic field, this equivalence is lost. We also obtain that, independently of the intensity of the magnetic field, the Landau levels can always be destroyed by suitably tuning rotation.

The paper is organized as follows. In Section 2 we write out the Hamiltonian for an electron constrained to the surface of a rotating sphere in the presence of a magnetic field, obtaining the Schrödinger equation of the problem. In Section 3 we solve the Schrödinger equation, obtaining the eigenvalues and eigenfunctions for three cases. We first analyze the effects of the rotation and magnetic filed individually, and then solve the general case where both are considered. In Sec. 4 we obtain the semi-classical orbits in order to have a better understanding of the influence of the rotation and magnetic field in the dynamics of a charged particle constrained to move on the surface of a sphere. The paper is summarized and concluded in Section 5.

2. Electrons in a rotating sphere in the presence of a magnetic field

The inertial forces, Coriolis and centrifugal, are given, respectively, by

$$\vec{F}_{Cor} = 2\mu(\vec{v} \times \vec{\omega}) \tag{1}$$

and

$$\vec{F}_{Cen} = \mu(\vec{\omega} \times \vec{r}) \times \vec{\omega}, \tag{2}$$

where μ is the effective mass and $\vec{\omega}$ is the angular velocity. Similarly to the electromagnetic forces, these inertial forces enter the Schrödinger Hamiltonian as a vector and a scalar potential, respectively. The Hamiltonian is given by

$$H = \frac{1}{2\mu}(\vec{p} - 2\mu\vec{A}_{ine})^2 + \mu V_{ine}, \tag{3}$$

where

$$\vec{A}_{ine} = \frac{1}{2}(\vec{\omega} \times \vec{r}) \tag{4}$$

and

$$V_{ine} = -\frac{1}{2}(\vec{\omega} \times \vec{r})^2. \tag{5}$$

Thus, the Hamiltonian for a point charge subjected to a magnetic field in a rotating reference frame is given by

$$H = \frac{1}{2\mu}[\vec{p} + e\vec{A} - \mu(\vec{\omega} \times \vec{r})]^2 - \frac{\mu(\vec{\omega} \times \vec{r})^2}{2} \tag{6}$$

$$= \frac{1}{2\mu}[\vec{p} + e\vec{A}]^2 - \vec{\omega} \cdot [\vec{r} \times (\vec{p} + e\vec{A})], \tag{7}$$

where $\vec{A}$ is the electromagnetic vector potential.

Even though the rotation can enter the Hamiltonian as a vector and scalar potentials, there is no gauge choice for the inertial vector potential (4) since it is not a gauge field. One can also note that the last term in Hamiltonian (7) does not breaks the gauge symmetry, since the transformation $\vec{A} \to \vec{A} - \vec{\nabla}\lambda(\vec{r})$, where λ is a scalar function of the coordinates, adds only a phase to the wavefunction. So, in order to have a more clear comparison between the effects of the rotation and the magnetic field, we use the symmetric gauge, given by

$$\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r}), \tag{8}$$

where $\vec{B}$ is the magnetic field.

It is important to mention that earlier studies of quantum particles constrained to move on a curved surface [36,37] revealed the appearance of a geometrical potential given by

$$V_g = -\frac{\hbar^2}{2m}(M^2 - K) = -\frac{\hbar^2}{8m}(k_1 - k_2)^2, \tag{9}$$

where M and K are the mean and Gaussian curvatures, respectively, and k_1 and k_2 are the principal curvature of the surface. Since the two principal curvatures in the sphere are equal, in our case this potential vanishes.

We consider that the charged particle is constrained to the surface of a sphere of radius R under the action of both a uniform magnetic field and a uniform rotation with corresponding axes in the z direction, so that $\vec{B} = B\hat{z}$ and $\vec{\omega} = \omega\hat{z}$.

In spherical coordinates we have that

$$\vec{B} = B(\cos\theta\hat{r} - \sin\theta\hat{\theta}). \tag{10}$$

Thus, the electromagnetic vector potential can be chosen to be

$$\vec{A} = \frac{RB\sin\theta}{2}\hat{\phi}. \tag{11}$$

The Hamiltonian becomes then

$$H = \frac{p^2}{2\mu} - \alpha R\sin\theta p_\phi + \beta\sin^2\theta, \tag{12}$$

where

$$\alpha = -\frac{eB}{2\mu} + \omega \tag{13}$$

and

$$\beta = \frac{eR^2B}{2}\left(\frac{eB}{4\mu} - \omega\right). \tag{14}$$

The Schrödinger equation is therefore given by

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + i\hbar\alpha\frac{\partial\psi}{\partial\phi} + \beta\sin^2\theta\psi = \mathcal{E}\psi. \tag{15}$$

With the *ansatz* $\psi = u_m(\theta)e^{im\phi}$, Eq. (15) becomes

$$u_m'' + \cot\theta u_m' + \left(E + \alpha'\hbar m - \beta'\sin^2\theta - \frac{m^2}{\sin^2\theta}\right)u_m = 0, \tag{16}$$

where $m = 0, \pm1, \pm2, \pm3, \ldots$ and

$$E = \frac{2\mu R^2}{\hbar^2}\mathcal{E}, \qquad \alpha' = \frac{2\mu R^2}{\hbar^2}\alpha, \qquad \beta' = \frac{2\mu R^2}{\hbar^2}\beta.$$

Using the substitution $x = \cos\theta$, the above equation becomes

$$\left[\frac{d}{dx}(1 - x^2)\frac{d}{dx} + \mathcal{E}' + \alpha'\hbar m - \beta'(1 - x^2) - \frac{m^2}{1 - x^2}\right]u_m = 0. \tag{17}$$

Please cite this article in press as: J.R.F. Lima et al., Effects of rotation on Landau states of electrons on a spherical shell, Phys. Lett. A (2018), https://doi.org/10.1016/j.physleta.2018.07.029